

This material is confidential until the end of November 1997.

## Directions

Test period                      May 12 - June 2 1997.

Test time                         180 minutes without a break.

Resources                        Calculator (not symbolic computation) and table of formulas.

Test material                    Test material should be handed in together with your solutions.

Write your name, gymnasium programme/adult education and date of birth on all the papers you hand in.

Test                                 The test is made up of 13 questions.

Most of the problems are of long-answer type. With these problems, it is not enough to give a short answer, it requires:

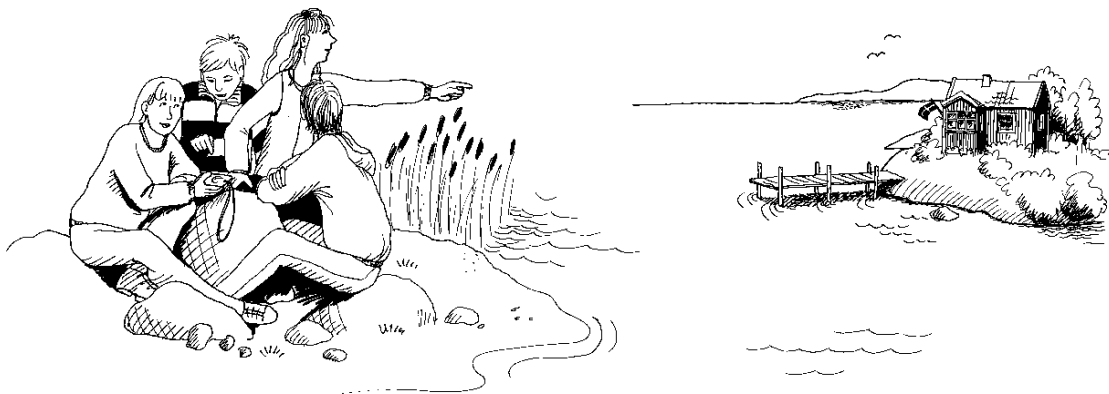
- that you write down what you do
- that you explain your train of thought
- that you draw figures when needed
- that you show how you use your calculator in numerical and graphical problem solving.

For some exercises, (where it says "*Only an answer is required*") only the answer needs to be given.

Try all of the problems. It can be relatively easy, even towards the end of the test, to earn some points for a partial solution or presentation.

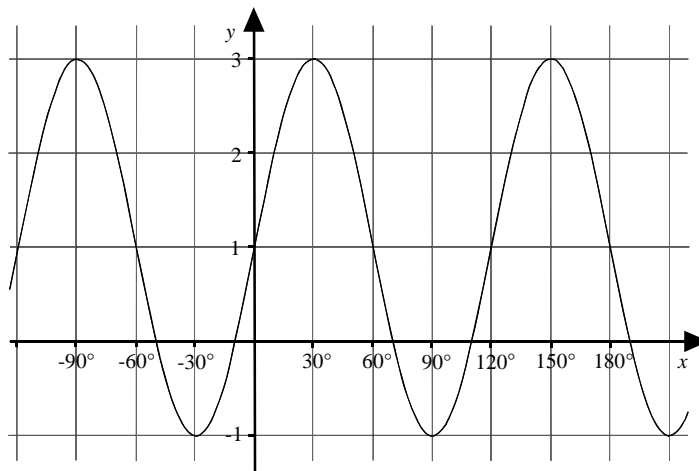
The grading levels              The teacher responsible will explain the grade which are required for "Passed" and "Passed with Distinction". On the test one can attain a maximum of 47 points.

1. Find the indefinite integral  $F(x)$  for the function  $f(x) = 3x^2 + 2x - 3$  if  $F(1) = 4$ . (2p)
  
2. Give an expression representing the area between the curve  $y = 4x - x^2$  and the  $x$ -axis. Calculate the area. (2p)
  
3. The integral  $\int_1^2 x(x-3)dx$  has the value  $-\frac{13}{6}$ . Verify this answer using an indefinite integral. (3p)
  
4. Some teenagers sat on a stone,  $S$ , on the beach and looked at a bridge,  $\ddot{O}$ , on an island far out in the bay. They decided to use their mathematical knowledge to calculate the distance between the stone and the bridge. They measured a distance  $SP$ , 100 m long, along the beach. After the measurement they estimated the angles  $SP\ddot{O}$  and  $PS\ddot{O}$  using a compass. The angles  $SP\ddot{O}$  and  $PS\ddot{O}$  were  $30^\circ$  and  $135^\circ$  each. What did they found out? (3p)



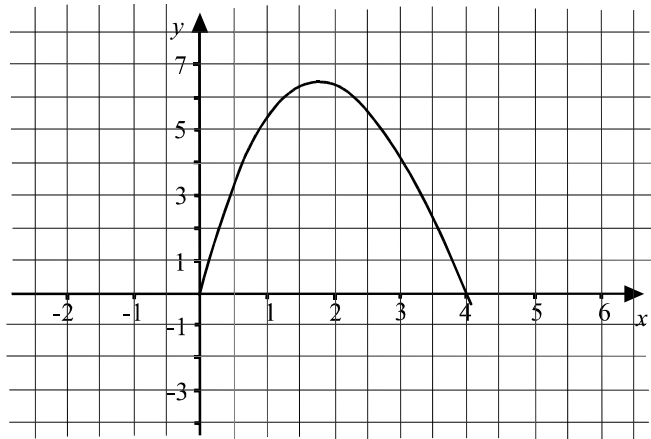
5. Simplify  $f(x) + f''(x)$  for the function  $f(x) = 4 \sin 3x - 5 \cos x$  (3p)

6. The diagram shows the graph of the function  $y = A \sin kx + b$   
 Find the constants  $A$ ,  $k$  and  $b$ . (Only answer is required) (3p)



7. Determine if  $y = x(\ln x - 1)$  is a solution of the differential equation  
 $y' = \frac{y}{x} + 1$  for  $x > 0$ . (3p)

8. In the figure to the right, the graph of the function  $y = f(x)$  is sketched.



One of the following alternatives

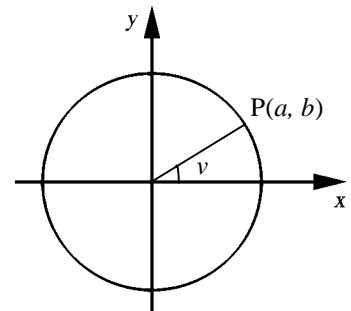
28, 24, 17, 12 and 6.3

is the correct value of the integral

$$\int_0^4 f(x) dx.$$

- a) Indicate which one of the above values is correct. (Only answer is required) (1p)
- b) Explain your choice in a). (2p)

9. The point P in a unit circle has the coordinates  $(a, b)$  (see figure to the right).  
Make a new figure and indicate where the angles  $v + 180^\circ$  and  $v + 270^\circ$  are situated on the unit circle.



Give the following trigonometric expressions in another way using the coordinates of the point P.

- a)  $\sin(v + 180^\circ)$
- b)  $\cos(v + 270^\circ)$  (3p)

10. Find the area of the region bounded by the curve  $y = \sqrt{2x + 3}$ , the line  $y = x$  and the  $x$ -axis. Express your answer in exact form. (4p)

11. a) Show that the equation  $\frac{2 \sin 2x}{1 - \sin^2 x} = 5$  can be written as  $\tan x = 1.25$ . (2p)
- b) Solve the equation  $\tan x = 1.25$  completely. (2p)

12. An electric engineer has programmed an automatic switch. The program follows a mathematical model which indicates the moment,  $M$ , during the day when it begins to get dark in a certain village:

$$M = 19 - 4 \cos\left(\frac{\pi(360 - t)}{180}\right)$$

where  $M$  is the time in hours ( $M = 12.5$  corresponds to 12:30) and  $t$  is the time in days ( $t = 1$  corresponds to January the first). The model works provided that each month is assumed to be 30 days long.

Find according to the model:

- at what time of the day it gets dark in the middle of April, (2p)
- for which months, are there days, when the darkness arrives at 6 p.m.(18:00), (3p)
- at which times of the year, the change from sunshine to darkness is the fastest. (3p)



13. The curve  $y = 4.5 \cdot e^{0.25x}$  and the line  $y = 12 - x$  enclose together with the  $x$ -axis and the  $y$ -axis an area. When this area is rotated about the  $x$ -axis, it generates a volume of revolution. Give a good approximation of this. (6p)