Concerning test material in general, the Swedish Board of Education refers to the Official Secrets Act, the regulation about secrecy, 4th chapter 3rd paragraph. For this material, the secrecy is valid until the expiration of November 2000.

Directions

Test time   180 minutes without a break.

Resources   Graphic calculator and table of formulae.

Test material   The test material should be handed in together with your solutions.

Write your name, the name of your education programme/adult education, and your date of birth on all the sheets of paper you hand in.

The test   The test consists of 14 problems.

Most of the problems are of the *long-answer type*. In these problems it is not enough to give short answers, they require
- that you write down what you do
- that you explain your train of thought
- that you, where necessary, draw figures
- that you show how you have used your resources when you have solved problems numerically/graphically

For some problems (where it says *Only an answer is required*) you only need to give the answer.

NOTE! The decimal point is placed at the bottom of the line i.e. $3.5 = 3\frac{1}{2}$

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for a partial solution or presentation.

The score levels   The teacher responsible will inform you about the scores required for "Passed" and "Passed with Distinction". The maximum score is 38 points.

1. The triangle ABC is given according to the figure below. Calculate the length of side AC.

C

(cm)

52°   42°

A
6,6
B
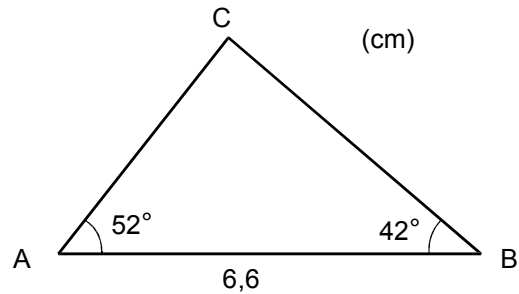
(2p)

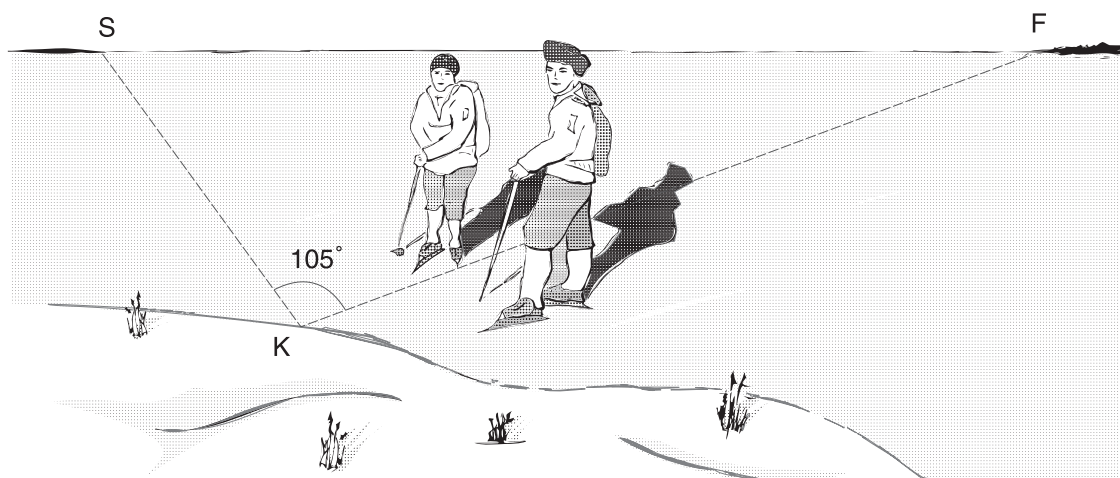2. Determine $y = F(x)$ if it is the integral of $f(x) = 8x^3 - 2x$ and $F(2) = 4$ (2p)

3. Find all solutions to the equation $\sin x = 0.6$ in the interval $0° < x < 450°$ (2p)

4. Determine the indefinite integral $\int \sin 2x \, dx$ and hence evaluate the

integral $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$ (2p)

5. Express as an integral the area bounded by the curves $y = 3x^2$ and $y = 16 - x^2$ and calculate the area. (3p)

6.

One calm, sunny winters day, Helen and Lotta are out long-distance skating. At 12.00 noon they arrive at Kappelskär. They know that it takes 35 minutes to skate from Kappelskär to Sundskär, and that it takes 60 minutes to skate from Kappelskär straight to Furusund. The bus from Furusund leaves at 14.30.

The angle between the lines of sight towards Sundskär and towards Furusund is estimated to 105°. They decide on skating to Sundskär, having some coffee and then skating straight from Sundskär to Furusund. How long a break can they take and still be able to catch the bus that leaves at 14.30?
We assume that Helen and Lotta skate at a constant speed.

(3p)

7. The temperature in a lake was measured during a cloudy summers day. The temperature turned out to follow the function $y(t) = 15 + 2\sin 0.26t$ where t is the number of hours after 12.00 noon.

a) Find $y'(t)$ *Only an answer is required*   (1p)

b) Find $y'(10)$ *Only an answer is required*   (1p)

c) Interpret what $y'(10)$ means with regard to the temperature of the water.   (1p)

8. Show that $y' - \dfrac{2y}{x} = x^2 \cos x$, when $y = x^2 \sin x$ (2p)

9. Let $g(x) = \int_0^x \frac{1}{1+t^2}\,dt$

   a) Interpret with a figure what $g(3)$ may mean. (2p)

   b) By using your calculator, find an approximate value of $g(3)$.

      Only an answer is required (1p)

10. Show how the relation $\cos 2A = 2\cos^2 A - 1$ may be obtained using the identities
    $\cos(u+v) = \cos u \cos v - \sin u \sin v$ and $\sin^2 u + \cos^2 u = 1$ (2p)

11. Find the positive constant A of the function $f(x) = 5 + A\sin 3x$ so that the
    maximum value of the function is twice as large as its minimum value. (2p)

12. A marble is dropped above a surface of water. The graph below shows how the
    velocity of the marble v m/s varies with time t seconds from the moment it is
   dropped.
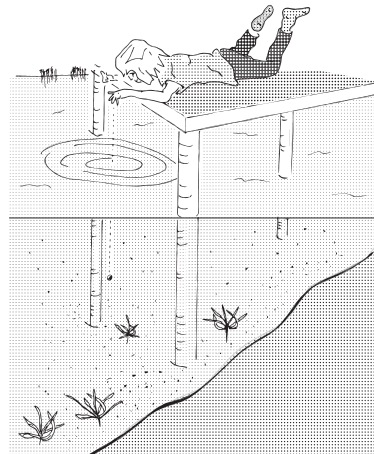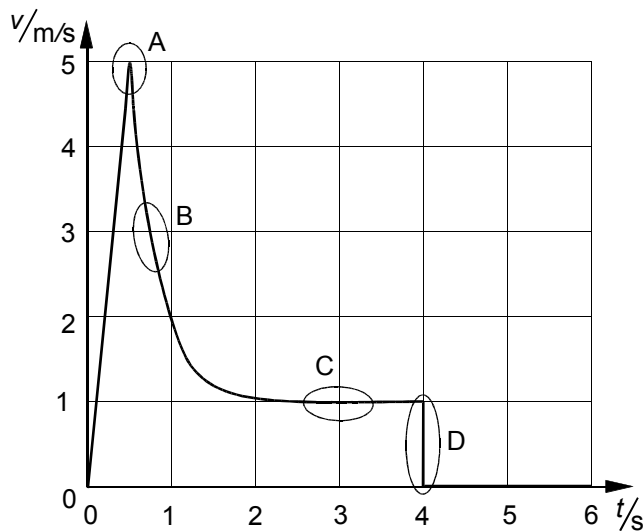
   a) Describe what happens to the marble in the regions A, B, C and D
      on the graph. (2p)

   b) How far above the surface of the water was the marble dropped? (1p)

   c) The velocity $v(t)$ m/s of the marble in the water can be described by the
      function $v(t) = 1 + 18e^{-3t}$. Find the depth of the water where the marble is
      dropped. Give the answer in metres and correct to two decimal points. (2p)

13. The figure below shows a square and the graph of a function. Choose a trigonometric function whose graph is similar to the one in the figure and calculate the area of the square for the function you have chosen. (3p)
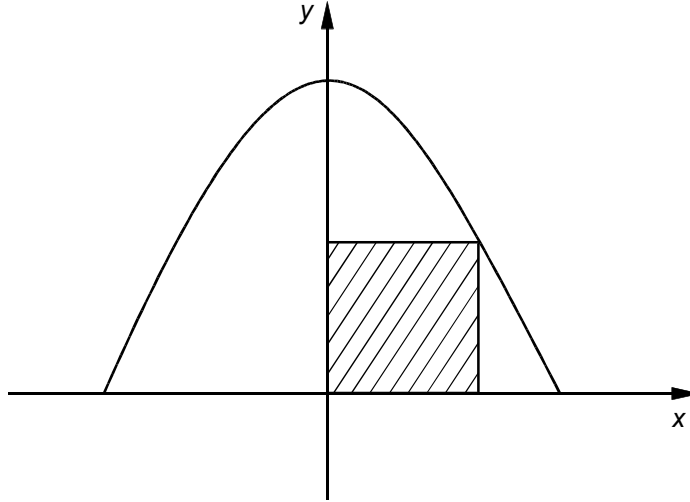
14. The functions f and g are differentiable.

A new function is formed $h(x) = (f(x))^2 + (g(x))^2$

For the functions f and g it is true that

- $f(0) = 2$ and $g(0) = 1$
- $f'(x) = g(x)$ and $g'(x) = -f(x)$

Find $h'(x)$ and use the result to show that $h(x) = 5$ for all x. (4p)