This material is confidential until the end of April 1998.

Directions

Test period	November 28 - December 18 1998.
Test time	240 minutes.
Resources	Part I: A collection of formulas Part II: A non-symbol manipulating graphics calculator and a collection of formulas.
Test material	This test paper should be handed in with your solutions. The solutions to part I is to be handed in before you may have access to your calculator. Therefore you must use a separate sheet of paper for your solutions to part I. Note that you may begin your work on part II without your calculator.
Test	The test is made up of 15 problems. Most of the problems are of the long-answer type. With these problems, it is not enough to give a short answer, it requires • that you write down what you do • that you explain your train of thought • that you draw figures when needed • that you show how you use your calculator in numerical and graphical problem solving. Try all of the problems. It can be relatively easy, even towards the end of the test, to earn some points for a partial solution or presentation.
The score levels	The teacher responsible will explain the scores which are required for ”Passed” and ”Passed with Distinction”. On the test one can attain a maximum of 55 points.

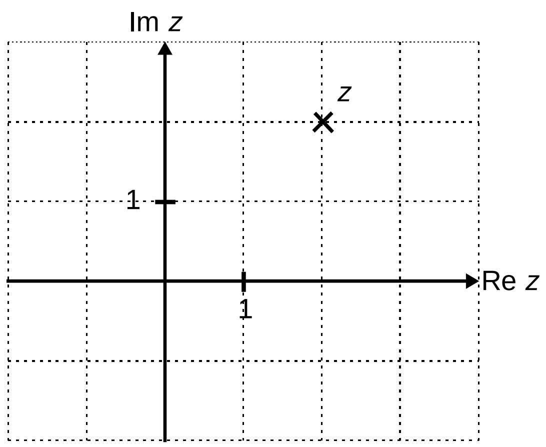
DEL I

This part contains 9 problems, which should be solved without a calculator. You must do your solutions to this part on a separate sheet of paper that will be handed in before you may have access to your calculator.

1. Solve the equation $8z - z^2 = 25$ (2p)

2. Calculate |*z*| when $z = \dfrac{1+2\mathrm{i}}{\mathrm{i}}$ (2p)

3. The number *z* is shown in the complex plane below.
Calculate $\dfrac{z}{\bar{z}}$ (3p)



4. The differential equation $y'' + 8y' - 9y = 0$ has many solutions.
Find one solution. (2p)

5. Solve the differential equation

a) $y' = \sin 2x$ if $y(0) = 2$ (2p)

b) $3y' - 2y = 0$ if $y'(0) = 5$ (2p)

6. Given that $z + 2\bar{z} = 1 + \mathrm{i}$, find z. (3p)

7. Show that $x^4 - x^2 \geq -0{,}25$ (3p)

8. Let $z_1 = 1 + \mathrm{i}$ and $z_2 = -\mathrm{i}$

a) Express each of the complex numbers in polar form. (2p)

b) Calculate the argument of $\dfrac{z_1^{\,4}}{z_2^{\,3}}$ (2p)

9. Calculate $|e^{2+\mathrm{i}}|$ (2p)

PART II

This part contains of 6 problems, which are intended to be solved with the aid of a graphics calculator (non-symbol manipulating). Note that you may begin your work on part II without your calculator.

10. a) Draw a complex plane and sketch the position of $z_1 = 3 + 4i$. (1p)

b) The complex numbers z_1 och iz_1 form a triangle with origo.
Calculate the area of the triangle. (2p)

c) Use the numbers *a* and *b* to form a general expression for the area of the triangle formed by the numbers *z*, iz and origo when $z = a + bi$. (2p)

11. You are a crewmember on a space-craft which is going to land on an unknown planet. At 06.34.15 the craft enters the atmosphere of the planet and at that time its speed is 6875 m/s according to the craft´s computer. Based on an analysis of the atmosphere the speed *y* m/s is assumed to decrease by the time *x* s in accordance with $\frac{dy}{dx} = -0.00075y$.
The slow-down of the space-craft is accomplished by retro-rockets. These rockets are to be ignited when the speed is 1500 m/s, but due to malfunctioning math-processor the time for ignition cannot be calculated by the computer. It must therefore be calculated by hand.
At what time should you ignite the retro-rockets? (4p)

12. In a factory, yeast is manufactured in a tank. The yeast has a growth rate propotional to its mass *y* kg with the proportionality constant 0.003 min^{-1}. When the process begins there is 200 kg of yeast in the tank.

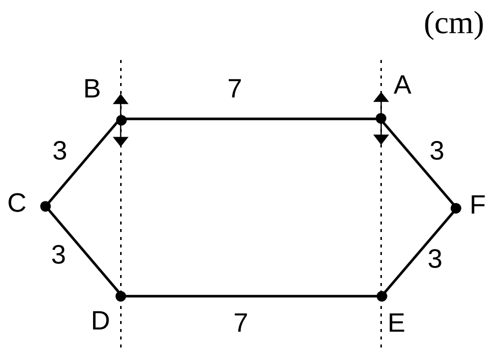
a) Find a differential equation that describes the growth rate of the yeast. (1p)

b) Use the model to find out how much yeast there ought to be in the tank after five hours? (3p)

c) During the production a constant flow of yeast mass is drawn off. Find a differential equation that describes the rate of change of yeast mass when *a* kg mass per minute is drawn off. (1p)

d) How much yeast can be drawn off per minute if the yeast mass should be at a constant level of 200 kg? (2p)

13. In the hexagon below the area can be varied by raising or lowering the side AB if the side DE is fixed and the lengths of all sides are constant. AB and DE are at all times parallel to each other.
Find the maximum area of the hexagon, with two significant digits. (4p)

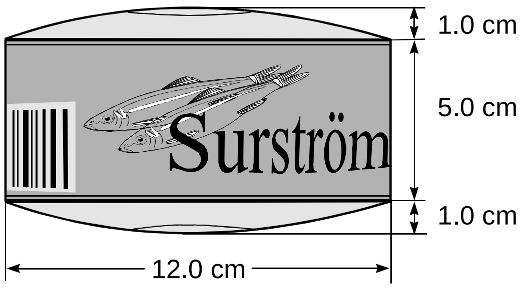


14. If the population growth rate is p % per year a rule of thumb states that it takes $\frac{70}{p}$ years to double the population.

a) Find the differential equation describing the population growth rate. (1p)

b) Use the differential equation to show that the rule of thumb is correct. (2p)

15. A beautiful spring evening Anders, a lover of fermented Baltic herring, plans to enjoy the contents of a can he bought last summer. During the winter the lid and bottom of the can has began to bulge due to the fermentation of the contents. From the beginning the can looked like a straight circular cylinder with a diameter of 12.0 cm and a height of 5.0 cm, but now it is a body that looks like the can in the figure below.



Anders observes that the ”profile” of the lid and bottom fairly well can be described by the graph of a polynomial function of the second degree ($y = ax^2 + bx + c$).

Calculate the increase of the volume in per cent, when the can bulges 1.0 cm on each side and the diameter and height are constant. (7p)