This material is confidential until the end of April1999.

Directions

Test period	December 1 – December 17 1998.
Test time	240 minutes.
Resources	Part I: A collection of formulas. Part II: A non-symbol manipulating graphics calculator and a collection of formulas.
Test material	This test paper should be handed in with your solutions. The solutions to part I should be handed in before you may have access to your calculator. Therefore you must use a separate sheet of paper for your solutions to part I. Note that you may begin your work on part II without your calculator. Write your name and the name of your education programme/adult education on all the sheets of paper you hand in.
The test	The test consists of 15 problems. Most of the problems are of the *long-answer type*. In these problems it is not enough to give short answers, they require • that you write down what you do • that you explain your train of thought • that you, where necessary, draw figures • that you show how you have used your resources when you have solved problems numerically/graphically For some problems (where it says *Only an answer is required*) you only need to give the answer. Try all of the problems. It can be relatively easy, even towards the end of the test, to earn some points for a partial solution or presentation.
The score levels	The teacher responsible will inform you about the scores required for ”Passed” and ”Passed with Distinction”. The maximum score is 50 points.

PART I

This part consists of 8 problems and is intended to be carried through without the use of a calculator.
Your solutions to this part should be presented on separate sheets of paper which must be handed in before you get access to a calculator.
Please note that the work with Part II can be started without the calculator.

1. Find the argument and the modulus of the complex number $-7+7i$
Only an answer is required (2p)

2. Write the expression $\dfrac{(3-i)(3+i)}{1+2i}$ in the form $a+bi$ (2p)

3. Find the function $y=f(x)$ that satisfies the conditions $f(1)=1$ och $y'=4x$ (2p)

4. Solve the differential equation $y''-6y'-7y=0$ (2p)

5. Eva and Martin have solved the same problem. Below, you can see how they did it.

a) How may the problem have been formulated? (2p)

b) Both Eva's and Martin's solutions are correct.
Why do they not receive the same answer? (2p)

Eva's solution:

$y'=y+x \quad y(1)=2$

$h=0.1$

$y(1.1)=2+0.1(2+1)=2.3$
$y(1.2)=2.3+0.1(2.3+1.1)=2.64$
$y(1.3)=2.64+0.1(2.64+1.2)=3.024$
Answer: $y(1.3)\approx 3.0$

Martin's solution:

$y'=y+x \quad y(1)=2$

$y_h=Ce^x$
$y_p=ax+b \ , \ y'_p=a$

$a=ax+b+x$
$a=-1 \qquad b=-1$
$y=Ce^x-x-1$
$y(1)=Ce-1-1=2$
$Ce=4$
$C=4e^{-1}$
$y=4e^{x-1}-x-1$
$y(1.3)\approx 3.099$

Answer: $y(1.3)\approx 3.1$

6. Find two different non-real numbers whose product is $-2+2i$ (2p)

7. When the solutions to the equation $z^3 - 4z^2 + 5z = 0$ are represented as points in the complex plane, it is possible to draw a circle that passes through all the points.

a) Solve the equation. (2p)

b) Find the radius of the circle. (2p)

8. Together with the *x*-axis, the line through the points P = (1, 0) and Q = (4, 2) and the curve to $y = \sqrt{x}$ form a region in the first quadrant. When the region is rotated about the *x*-axis, a solid of revolution is generated. Show that the volume of the solid of revolution is 4π units of volume. (4p)

PART II

This part consists of 7 problems and is intended to be carried through with a calculator (graphic but not with symbolic manipulation).
Please note that the work with Part II can be started without the access to a calculator.

9. Solve the differential equation $y' + 2y = 0$ when $y(1) = 1$ (2p)

10. a) Draw a complex plane and mark the number $z_1 = -4 + 3i$

Only an answer is required (1p)

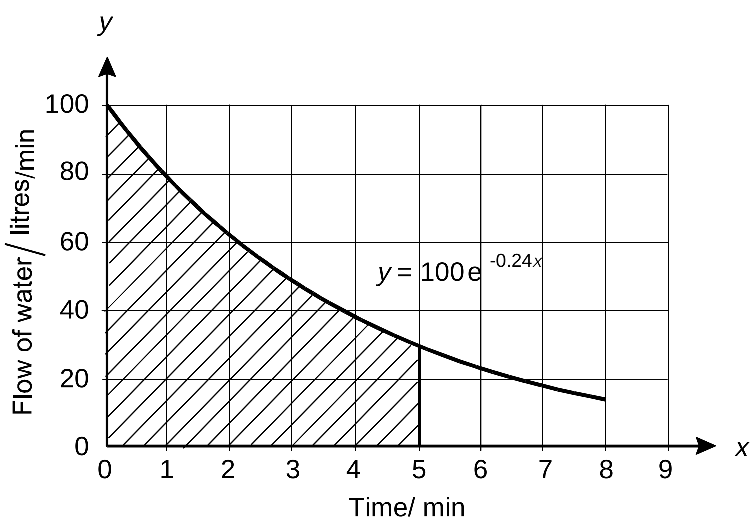
b) Together with the origin, the points representing z_1 and $\bar{z}_1$ form corners in a triangle. Find the area of the triangle. (2p)

c) Write down a general expression in *a* and *b* of the area of the triangle formed by z, $\bar{z}$ and the origin, if $z = a + bi$ (3p)

11. A water tank contains 2000 litres of water. By a leak in the tank, *y* litres/min of water starts to flow out, where $y = 100e^{-0.24x}$ and *x* minutes is the time since the water started leaking.

a) In the figure below, the graph to the function $y = 100e^{-0.24x}$ is drawn. Calculate the shaded area under the curve. (2p)

b) Interpret what this area means in relation to the example with the water tank. (1p)



12. At a check-up, there were 250 snails in a garden. One week later, the number was 268. We assume that the rate of change is proportional to the number of snails.

a) Let y be the number of snails after x weeks and write down a differential equation that describes the rate of change in the number of snails.

Only an answer is required (1p)

b) Solve the differential equation and calculate how many snails there will be in the garden in three weeks. (3p)

13. One morning, around eight o'clock, a lake started to freeze over. The thickness of the ice y cm is a function of the time t hours after the water started to freeze. The growth rate of the ice can be described by the differential equation

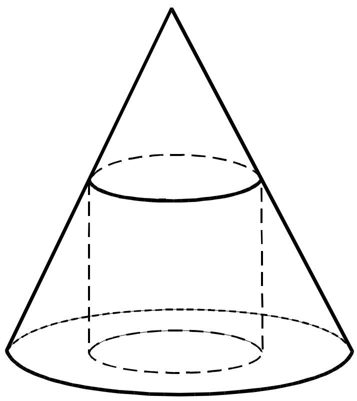
$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{k}{y}$, where k is a constant.

a) The solutions to the differential equation can be written as $y^2 = 2kt + C$
Show that y is a solution to the above differential equation. (2p)

b) After one hour, the thickness of the ice was 1 cm. To be able to skate on the ice, the thickness has to be at least 3 cm. How soon is it possible to start skating on the ice? (3p)

14. On a table, there is a straight circular cylinder with radius R = 3.0 cm and height H = 6.0 cm. A straight circular cone will be made to the cylinder. The cone will be placed over the cylinder so that the base of the cone rests on the table.

Find the smallest possible volume of the cone. Give the answer to two decimal places.



(4p)

15. Bismuth decays into thallium at a decay-rate of 32% per minute. Thallium decays into lead at a decay rate of 15% per minute. The lead that is formed is stable and therefore it does not decay.

At a laboratory test, we start from a certain amount of pure bismuth. We denote the quantity of bismuth A and the quantity of thallium B, at the time when the decay has proceeded for t minutes. The rate of change in the amount of thallium can then be described by the differential equation $\frac{\mathrm{d}B}{\mathrm{d}t} = 0.32A - 0.15B$

a) Denote the amount of lead by C and form differential equations that describe the rates of change in the amounts of bismuth and lead.

Only an answer is required (2p)

b) What can you say about the rates of change in the amounts of bismuth, thallium and lead at the time when the amount of thallium is at its maximum? (2p)

Background: The problem with radon

At the radioactive decay of gaseous radon, substances are formed which in turn decay, so-called radon daughters. Among these substances are bismuth, thallium and lead. They can go down the lungs and because of their radiation, they increase the risks of lung cancer. Some scientists think that at least one tenth of all deaths in lung cancer in Sweden are caused by inhalation of radon.

Radon from the ground is the most common cause of radon problems in dwellings. Radon is not found in all grounds, but for example in boulder-ridges in Bergslagen, in granite in some places in Norrland and in alum-shale. Some building materials, for example blue porous concrete, which is no longer produced, have turned out to give off radon and have thereby caused environmental problems in houses built of these materials. The problems with radon from the ground or from building materials can be taken care of by improved ventilation in the house.

In 1994, the National Institute of Radiation Protection estimated that 200 000 Swedish dwellings have a higher content of radon than the permitted limit. By then, 320 000 dwellings have had their content of radon measured, and 20 000 had been degasificated. At such a renovation, the Swedish state may give a contribution.
(Source: Bra Böckers Lexikon 2000, 1998)