Directions

Test time	Totally 240 minutes.
Resources	Part I: Table of formulae Part II: Calculator (graphic but not symbolic computation) and table of formulae.
Test material	The test material should be handed in together with your solutions. Solutions to Part I should be handed in before you get access to a calculator. You should therefore present your work on Part I on separate sheets of paper. Please note that the work with Part II can be started without the calculator. Write your name, the name of your education programme / adult education, and your date of birth on all the sheets of paper you hand in.
The test	The test consists of 17 problems. In most of the problems, it is not enough to give short answers, they require • that you write down what you do • that you explain your train of thought • that you, where necessary, draw figures • that you show how you have used your resources when you have solved problems numerically/graphically For some problems (where it says *Only an answer is required*) you only need to give the answer. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for a partial solution or presentation.
The score levels	The teacher responsible will inform you about the scores required for ”Passed” and ”Passed with Distinction”. The maximum score is 52 points.

Part I

This part consists of 9 problems and is intended to be carried through without the use of a calculator.
Your solutions to this part should be presented on separate sheets of paper that must be handed in before you get access to a calculator.
Please note that the work with Part II can be started without the calculator.

1. Let $z = -1 + \mathrm{i}$

a) Draw a complex plane and mark the number *z*. (1p)

b) Find $\arg z$ *Only an answer is required* (1p)

c) Find $|z|$ *Only an answer is required* (1p)

2. Given that $z = 2 - 2\mathrm{i}$, find $z\bar{z}$ (2p)

3. Solve the equation $\dfrac{z^2}{z-1} = 2$ (2p)

4. Solve the differential equation $y' + 2y = 0$ when $y(0) = 8$ (2p)

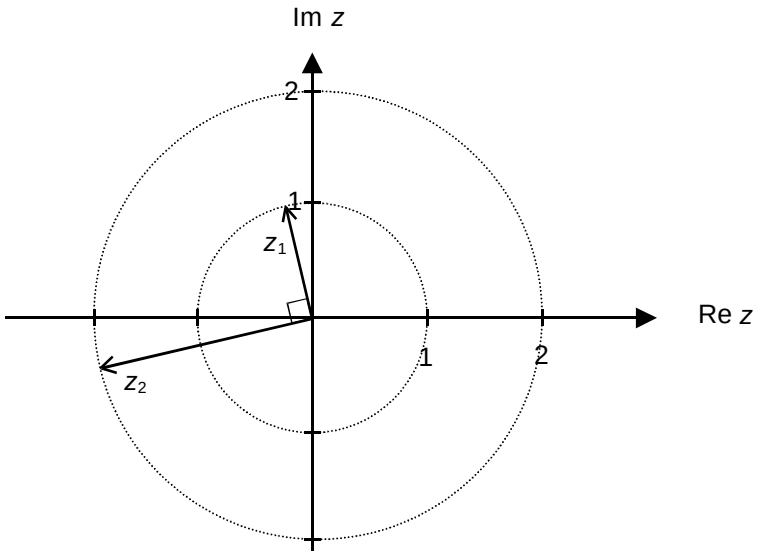
5. Sometimes in calculations with complex numbers, de Moivre's theorem is used which can be written $z^n = r^n(\cos nv + \mathrm{i}\sin nv)$

In your own words, describe the meaning of de Moivre's theorem. Your description should include the notions *argument*, *absolute value* and *polar form*. (2p)

6. Find a differential equation of the second order that has one solution $y = \mathrm{e}^x + \mathrm{e}^{-x}$ (2p)

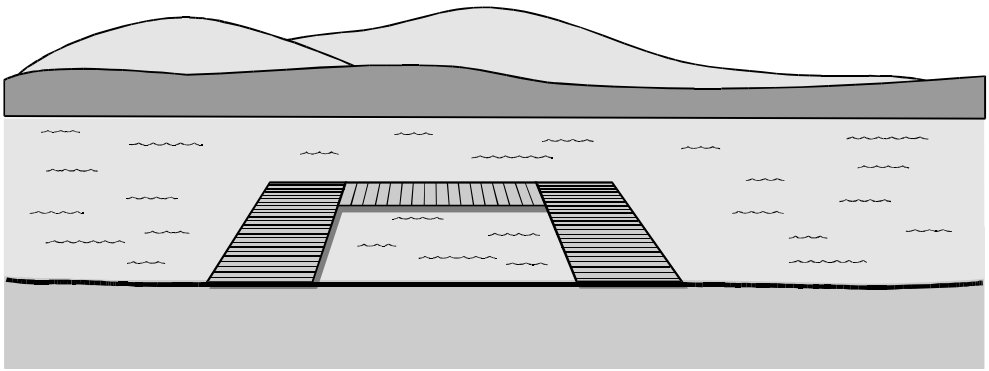
7. Given that $y' = y - x$ and $y(0) = 2$, find $y(1)$ with a *numerical method*. Use a step length of 0.5. (2p)

8. In the figure below, there is information about the complex numbers z_1 and z_2
Given that $zz_1 = z_2$, find z. Answer in the form $z = a + b\,\mathrm{i}$ (3p)



9. In a bay they want to build a rectangular jetty. It will consist of two pieces connected to the shore and a third piece that connects the outer edges. (see figure). All the pieces of the jetty are 1.0 metre wide.

What is the maximum area of the water surface inside the bridge, if the area of the bridge and the enclosed water surface is 128 m^2? (4p)



Part II

This part consists of 8 problems and is intended to be carried through with a calculator (graphic but not with symbolic computation). Please note that the work with Part II can be started without the access to a calculator.

10. Given that $z_1 = \sqrt{3} + 2\mathrm{i}$ and $z_2 = 2 + \mathrm{i}\sqrt{3}$, find $\dfrac{z_1}{z_2}$. (2p)

Answer in the form $a + b\mathrm{i}$ with exact values of *a* and *b*.

11. Solve the differential equation $y'' - 13y' = 0$, given that $y(0) = 4$ and $y'(0) = 13$ (3p)

12. In the complex plane, mark $z = t + (t-3)\mathrm{i}$ for some real values of *t*. Formulate a conclusion about the complex numbers *z*. (2p)

13. In Sweden, the number of peregrine falcons decreased from 1955 and onwards. The rate of the decrease in pairs/year is regarded as proportional to the number of peregrine pairs. When they started counting them in 1955 there were 88 pairs, but ten years later there were only 13 pairs left.

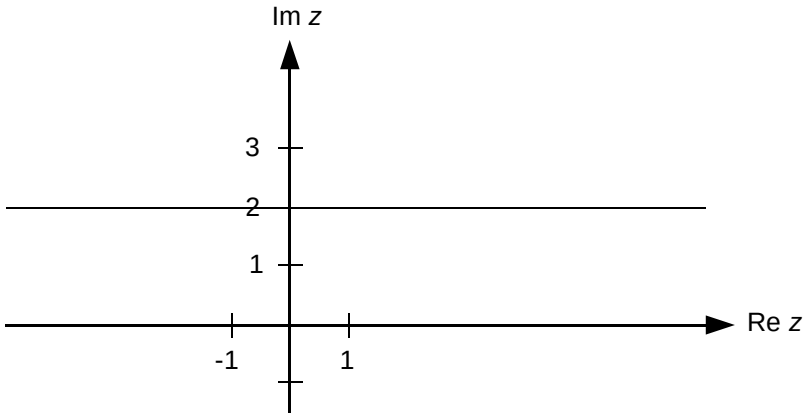
a) Formulate a differential equation that describes the situation, and solve it. (2p)

b) According to this model, when would the peregrine falcons have been extinct? (2p)

In the Swedish Society for the Conservation of Nature's (SNF) journal "Sveriges natur" no 4 1965 they wrote: "SNF's inventory on the peregrine falcon shows that the situation for this species is, if possible, even more disastrous: so far only 13 pairs have been reported from the whole country. Only 6 of these have had any nestlings! Since 1955, about 75 pairs have "disappeared" – and any other factor than death by biocide is hardly possible."

SNF's rescue work for the peregrine falcon has been very successful. Current information from the journal "Sveriges natur" says that the peregrine falcon has strongly recovered, and that in 1988 the population amounted to about 70 pairs. More than 130 nestlings were fully fledged this year.

14. The number z is located on the marked line in the complex plane below. What values can the real part of z^2 assume? (3p)



15. Two of the roots of the equation $z^3 - 2z^2 + az + b = 0$ are $z_1 = \mathrm{i}$ and $z_2 = -\mathrm{i}$ (a and b are real numbers).

a) Find the constants a and b. (2p)

b) Find the third root of the equation. (2p)

16. On a road, they lay a thin layer of tarmac which initially has a temperature of 120°C. The temperature of the tarmac y ºC is a function of time t min. The temperature of the air is 20 ºC. In a simple model, the tarmac is assumed to cool down with a speed proportional to the difference between the temperature of the tarmac $y(t)$ ºC and the temperature of the surrounding air. The constant of proportionality is $-0.046\,\mathrm{min}^{-1}$.

a) Write down a differential equation with initial conditions that describe the cooling down process. (2p)

b) Solve the differential equation and calculate how long time it takes for the tarmac to cool down to 30 ºC. (3p)

17. In the advertisement for a new car it is said that the acceleration during overtakings is remarkably good. The car is said to accelerate from 60 km/h (16.7 m/s) to 100 km/h (27.8 m/s) on 10 seconds on the fourth gear.
Suppose that the speed *v* m/s at different points of time during the acceleration can be described with the expression $v(t) = a\sqrt{t} + b$, where *a* and *b* are constants and *t* seconds is the time the car has accelerated.

a) Find the constants *a* and *b*. (2p)

b) According to this model, how long distance is needed for an overtaking that takes 10 seconds, if your initial speed is 60 km/h. (2p)

c) One such car is behind a lorry that drives 60 km/h and the driver is about to do an overtaking on a straight road with clear sight. How long time does the overtaking take if it is done according to the pictures below? (3p)