Directions

Test time	Totally 240 minutes.
Resources	Part I: Table of formulae Part II: Calculator (graphic but not symbolic computation) and table of formulae.
Test material	The test material should be handed in together with your solutions. The solutions to Part I should be handed in before you may have access to your calculator. You must therefore use a separate sheet of paper for your solutions to Part I. Note that you may begin your work on Part II without your calculator. Write your name, the name of your education programme / adult education, and your date of birth on all the sheets of paper you hand in.
The test	The test consists of 16 problems. In most of the problems, it is not enough to give short answers, they require • that you write down what you do • that you explain your train of thought • that you, where necessary, draw figures • that you show how you have used your resources when you have solved problems numerically / graphically For some problems (where it says *Only an answer is required*) you only need to give the answer. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for a partial solution or presentation.
Score levels	The teacher responsible will inform you about the scores required for “Passed” and “Passed with distinction”. The maximum score is 48 points.

Part I

This part consists of 10 problems and is intended to be carried through without the use of a calculator.
Your solutions to this part should be presented on separate sheets of paper that must be handed in before you get access to a calculator.
Please note that the work with Part II can be started without the calculator.

1. Let $z = 2 + 2\mathrm{i}$

a) Find $\bar{z}$ *Only an answer is required* (1p)

b) Find $|z|$ *Only an answer is required* (1p)

c) Write z on the polar form. *Only an answer is required* (1p)

2. Find the particular solution of the equation $y' - 4y = 0$ that satisfies the condition $y(0) = 5$ (2p)

3. Find the general solution of the differential equation $2y'' - 8y' + 8y = 0$ (2p)

4. Express the complex number $\dfrac{4\mathrm{i}}{1+\mathrm{i}} + \mathrm{i}$ in the form $a + b\mathrm{i}$ (2p)

5. Start with the numbers $z_1 = 1.5(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3})$ and $z_2 = 2(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6})$

a) Find $z = z_1 \cdot z_2$ (2p)

b) Point out $z = z_1 \cdot z_2$ in the complex plane. (1p)

6. The equation $z^3 - z^2 + 3z - 3 = 0$ is given.

a) Show that $z = 1$ is a root to the equation (1p)

b) Find the other roots to the equation. (2p)

7. The differential equation $y' = y + x - 2$ has one solution, which satisfies the condition $y(1) = 2.$ Use Euler's method with step size $h = 0.5$ and determine for this solution

a) $y(2)$ (2p)

b) $y(0.5)$ (1p)

8. Write down a differential equation on the form $y' = f(y)$ that has a general solution $y = A\mathrm{e}^{3x} + 4$ (2p)

9. Show that if $z_1 = a + b\mathrm{i}$ and $z_2 = a - b\mathrm{i}$ (*a* and *b* real) are roots to the equation $z^2 + pz + q = 0$ then *p* and *q* are real. (2p)

10. Show that $z \cdot \bar{z} \geq 4$ when $z = 2\sqrt{x} + \dfrac{1}{\sqrt{x}}\mathrm{i}$ and $x > 0$ (3p)

Part II

This part consists of 6 problems and is intended to be carried through with a calculator (graphic but not with symbolic computation). Please note that the work with Part II can be started without the access to a calculator.

11. Find the solution of the differential equation $y'' + 4y' - 5y = 0$ that satisfies the conditions $y(0) = 0$ and $y'(0) = 6$ (3p)

12. The air-pressure y kPa decreases with the height x km above the sea. On every height the air-pressure changes at a rate proportional to the present air-pressure.

a) Express this with a differential equation. (1p)

b) At the sea level, the air-pressure is 101 kPa. Find the constant of proportionality if the air-pressure has halved at the height 5.5 km. (2p)

13. Find z when $|z| = 4$ and $\operatorname{Re} z = -\operatorname{Im} z$ (3p)

14. Nisse has an aquarium that apart from fish, plants and stones contains 200 litres of water. Nitrate is formed in the water, and if the fish are to stay fit and well, Nisse has to change water in the aquarium.

Earlier, he regularly drew off 50 litres of water from the aquarium and replaced it with an equal amount of fresh water. If there were 100 mg of nitrate per litre of water before the water change, the concentration was 75 mg/l afterwards.

Now Nisse has equipped his aquarium with equipment for water change. It draws off water from the aquarium and at the same time pours in the same amount of fresh water through a tube. The water that comes from the aquarium will be well mixed, since the filter is switched on all the time, and the flow of the fresh water in the tube is not very powerful.

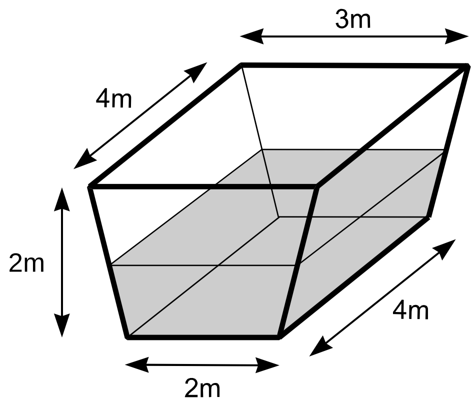
During the water change, the concentration of nitrate in the aquarium can be described by the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{y}{200}$ where y mg/l is the concentrate of nitrate when x litres of fresh water have been added.

First, help Nisse solve the equation, and then calculate how much water that has to be added according to the new method to reduce the concentration of nitrate from 100 mg/l to 75 mg/l. (3p)

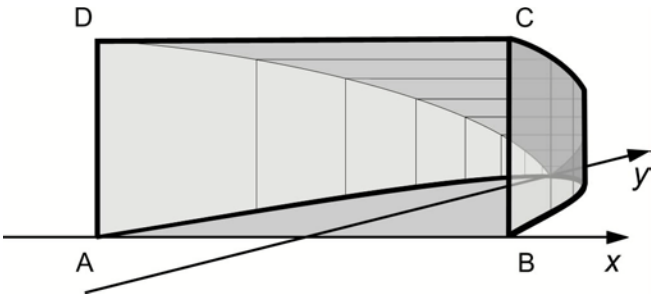
15. The figure below shows an open water tank. Water is filled into the tank at a constant rate of 2.4 m^3 per minute.

a) Write down a mathematical expression for the volume of water in the tank as a function of the depth of the water. (2p)

b) At what rate does the water surface rise when the depth is 0.50 metres? (2p)



16. An architect starts sketching on a draft of a concert hall. Her vision is that the floor should be limited by a quadratic curve (parabola) and a straight line at the back wall, perpendicular to the parabola's axis of symmetry. Each cross-section of the hall, parallel to the back wall ABCD, should be rectangular and have a height equal to half the width. The back wall should be 40 m wide and the concert hall is 64 m long. The architect wants to investigate the air-volume per person in the hall.



a) Write down the equation of the parabola. (1p)

b) Calculate the volume of the hall. (3p)

c) Calculate the area of the floor in the concert hall. What is the seating-capacity of the floor? Use this to estimate the air-volume per person. You have to do the assumptions needed to solve the problem. (3p)