Directions

Test period	May 9 - 15, 1996.
Test time	240 minutes without a break.
Resources	Calculator (not symbolic computation) and table of formulas.
Test material	Test packet and rough paper should be handed in when the test is completed, as well as the solutions. Write your name, gymnasium programme/adult education and date of birth on all of the papers you hand in.
Test	The test is made up of 14 problems. Most of the problems are long-answer type. With these problems, it is not enough with just a short answer, it requires • that you write down what you do and explain your train of thought • that you draw figures when needed • that you write down all of your computations. Try all of the problems. It can be relatively easy, even at the end of the test, to earn some points for a partial solution or presentation.
The grading levels	The teacher responsible will explain the grade levels which are required for ”Passed” and ”Passed with Distinction”. The test can earn a maximum of 50 points.

1. Write in polar form the complex number $3 + \mathrm{i}\sqrt{3}$. 2 p

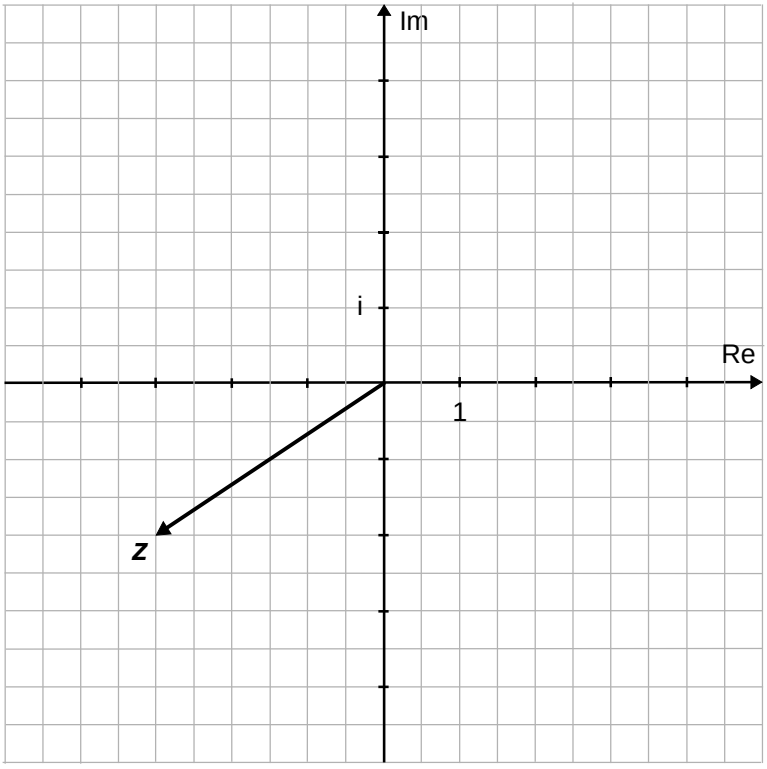
2. Express the complex number z in the form $a + b\mathrm{i}$. 2 p

$$z = \frac{4 - 3\mathrm{i}}{3 - 4\mathrm{i}}$$

3. Find the general solution of the following diffential equation. 2 p

$$y'' - 12y' + 32y = 0$$

4. The complex number z is represented in the figure.



Represent in the complex plane the number $1 - z$. 2 p

5. Solve the initial-value problem. 3 p

$3y' - 2y = 0\,;\qquad y'(0) = 5.$

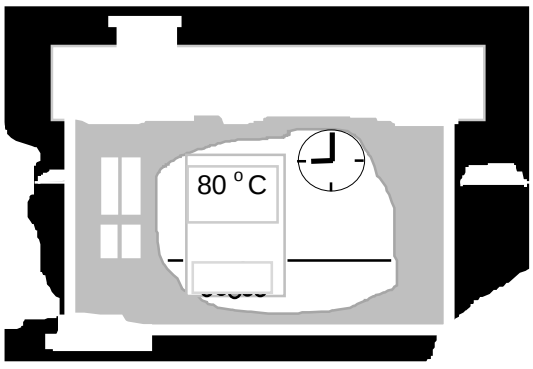
6. An electric current *y* measured in milliamperes flows through an inductor. The current depends on time *x* measured in seconds as follows

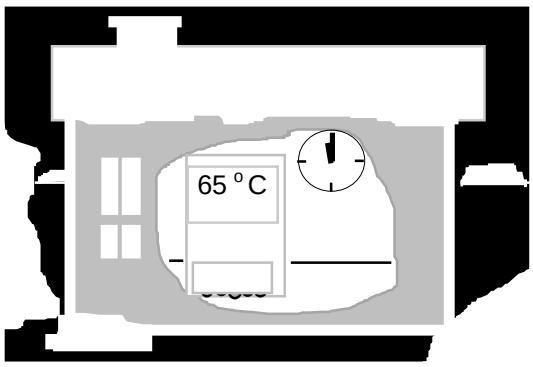
$y = 11.2 + 5.28x - 0.044x^2 \qquad \text{for} \quad 0 \le x \le 60$

At what time is the rate of change of current 2.5 milliamperes per second? 3 p

7. Find all roots of the equation $z^3 + 6z^2 + 11z = 0$. 3 p

8.





During wintertime a water heater is heated with firewood. The heater is in a storehouse where the air temperature is kept at 0 °C, at all times. When the firewood is used up and the heating ceases, the decrease in water temperature is proportional to its temperature *y*. One winter evening the firewood is all used up by 9 pm when the water temperature is 80 °C. At midnight the water temperature has gone down to 65 °C.

State a differential equation which describes the situation and decide when you must get up the next morning, if you want to take a shower using water with a temperature of 40 °C. 4 p

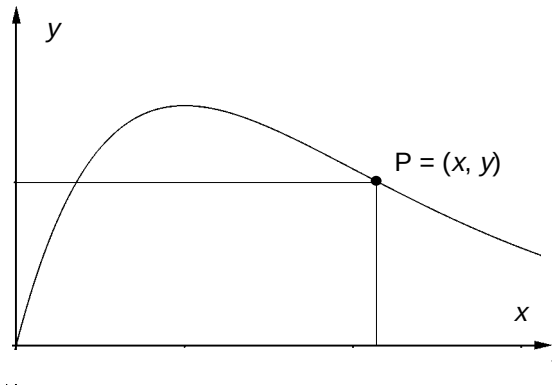
9. Choose real numbers k and m $(k \neq 0,\ m \neq 0)$ so that

$\int_0^2 (kx + m)\mathrm{d}x$ equals 0 with your choice of k and m. 2 p

10. Find appropriate initial conditions at $x = 0$ and constants a and b so that the initial-value problem $y'' + ay' + by = 0$ has a solution $y = 3\mathrm{e}^{-x} + 2\mathrm{e}^{2x}$.

4 p

11. A rough sketch of the graph of $y = 4x \cdot \mathrm{e}^{-x}$ is shown in the figure, for $x \geq 0$. From a point P on the curve, two lines are drawn to x- and to y-axis so that a rectangle is formed (see figure).



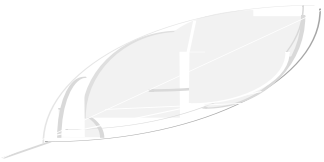
Verify using calculus that the area of the rectangle as a function of x has a local maximum at $x = 2$. 3 p

12. Given are two functions f and g with following properties

- $f(0) = 4$ and $f'(x) = 3 - 6\mathrm{e}^{-2x}$
- $g'(x) = f'(x)$
- the area of the region bounded by the graphs $y = g(x)$, $y = f(x)$ and the lines $x = -1$ and $x = 4$ equals 10.

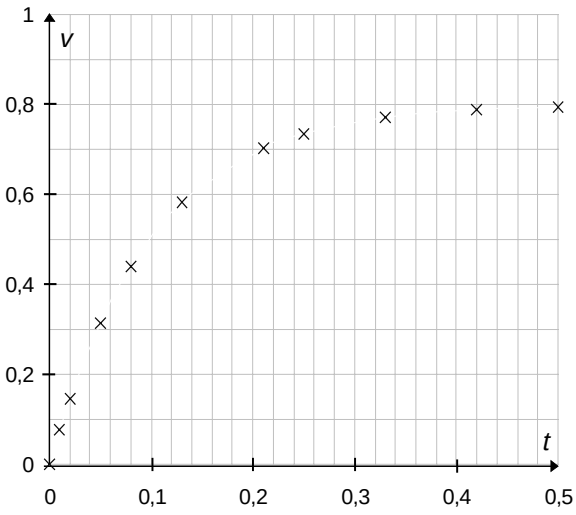
Find $g(x)$. 5 p

13.

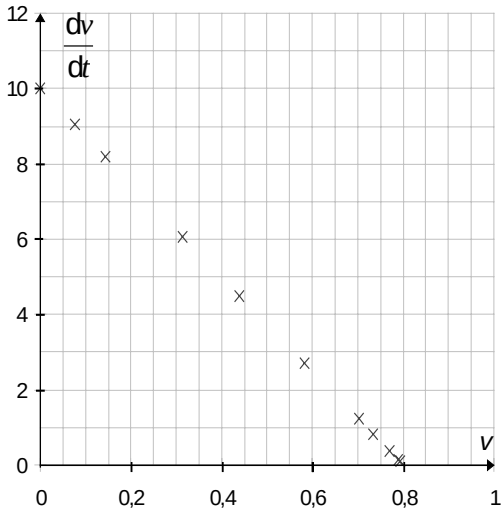


A feather is dropped from the height of 4.0 m. The motion of the feather is monitored and analysed by a computer. The results are presentated in the following two diagrams.

Velocity v in m/s as function of time t in seconds



Acceleration $\frac{dv}{dt}$ i m/s^2 as function of velocity v in m/s.



a) Describe the motion of the feather. 1p

b) Suggest an appropriate differential equation which describes $\frac{dv}{dt}$ depending on v . Solve this equation for v. 4 p

c) Use diagram 1 to estimate the distance, through which the feather has fallen during the first 0.50 seconds. Compare this result with the actual value implied by your differential equation. 3 p

14. One part of Mathematics is called complex analysis. In this you regard analytic functions $w = f(z)$ as a mapping from a domain in the *z*-plane onto a domain in the *w*-plane. See figure.

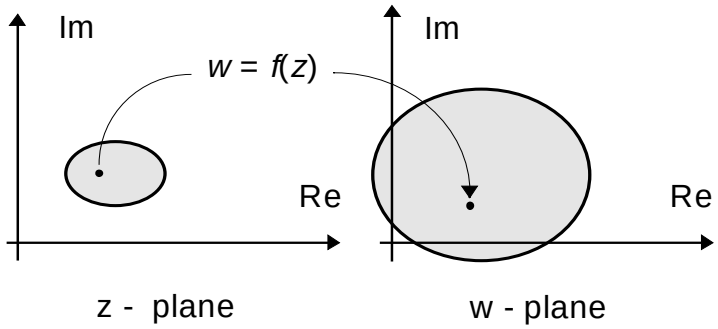


Figure.

The three complex numbers 0, 2 och $2 + 2i$ are vertices of a right angled triangle in the *z*-plane. Let $= z^2$ be a mapping of the triangle onto the w-plane.

a) Map the three vertices onto the *w*-plane. 1 p

b) Describe the mapping of the hypotenuse onto the *w*-plane. 2 p

c) Find the image of the other two sides of the triangle. 3 p