This material is confidential until the end of November 1997

Directions

Test period	April 21 - June 2 1997.
Test time	240 minutes without a break.
Resources	Calculator (not symbolic computation) and table of formulas.
Test material	The test material should be handed in together with your solutions.
	Write your name, gymnasium programme/adult education and date of birth on all the papers you hand in.
Test	The test is made up of 12 problems.

Most of the problems are of the long-answer type. With these problems, it is not enough to give a short answer, it requires

- that you write down what you do
- that you explain your train of thought
- that you draw figures when needed
- that you show how you use your calculator in numerical and graphical problem solving.

For some exercises, (where it says *”Only an answer is required”*) only the answer needs to be given.

Try all of the problems. It can be relatively easy, even towards the end of the test, to earn some points for a partial solution or presentation.

The score levels	The teacher responsible will explain the score levels which are required for ”Passed” and ”Passed with Distinction”. On the test one can attain a maximum of 69 points.

1. If $z = 2 + 2i$ and $w = 5i$,

a) Sketch the position of z, w and $z \cdot w$ in the complex plane (2p)

b) Express z and w in polar form (3p)

c) Evaluate $\arg\left(\frac{w}{z}\right)$ *Only an answer is required* (1p)

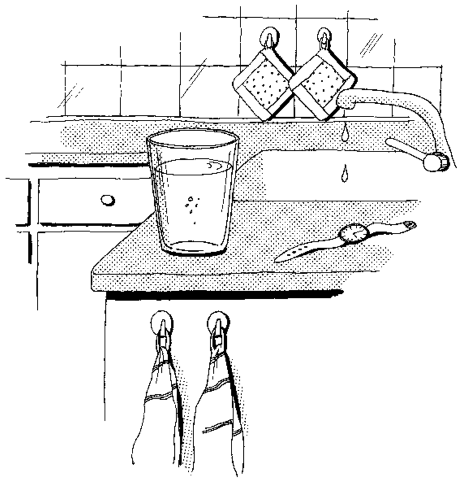
d) Calculate an exact expression for $|z \cdot w|$ *Only an answer is required* (1p)

e) Evaluate $1 - w \cdot \bar{z}$ (2p)

2. Solve the differential equations

$y' = 3x$ $\quad y' = 4y$ $\quad y'' = 5$ (5p)

3. A glas of cold water is placed in a room where the temperature is 20 °C. The differential equation $\frac{dy}{dt} = -0.1(y - 20)$ describes how the temperature y of the water increases, y is expressed in °C and the time t in minutes.
$y = 20 - 19e^{-0.1t}$ is one solution to the differential equation.



a) What was the initial temperature of the water? (1p)

b) Calculate the rate of change of the water temperature at the time when it is 10 °C. (2p)

c) Calculate the rate of change of the water temperature when 10 minutes have elapsed. (2p)

d) Per monitors the change of water temperature with a digital thermometer which shows the temperature in degrees Celcius as integers. According to his data, the water reaches room temperature after 36 minutes. Stina also measures the water temperature with a digital thermometer, but hers has an accuracy of tenths of degrees Celsius. Her data shows that it takes 59 minutes for the water to reach room temperature.
Explain why their results differ. (3p)

4. a) Develop $\left(2\sqrt{3}+2\mathrm{i}\right)^6$ (3p)

b) If $z=\left(2\sqrt{3}+2\mathrm{i}\right)^n$, find the whole numbers n for which $\operatorname{Re} z = 0$ (3p)

5. Solve the equation $x^3-4x^2+13x=0$ (3p)

6. a) Solve the differential equation $y''+y=0$, given that $y(0)=3$ and $y'(0)=0$ (3p)

b) Explain why y has a local maximum at $x=0$ (1p)

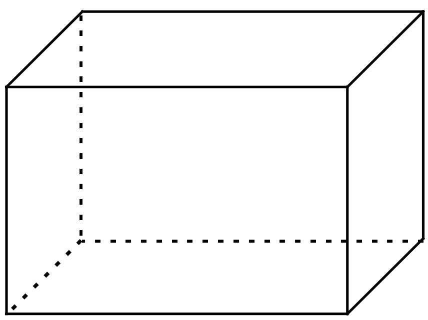
7. Find all combinations of the real numbers a and b for which the complex number $z=a+b\mathrm{i}$ satisfies $\bar{z}=z^2$ (4p)

8. Find the volume of the solid generated by the area enclosed by the line $y=2$ and the curve $y=6-x^2$ when it is rotated round the straight line $y=2$. (5p)

9. If $\operatorname{Re} z=5$ for a complex number z, find all possible values for $\operatorname{Re}\left(\dfrac{1}{z}\right)$. (4p)

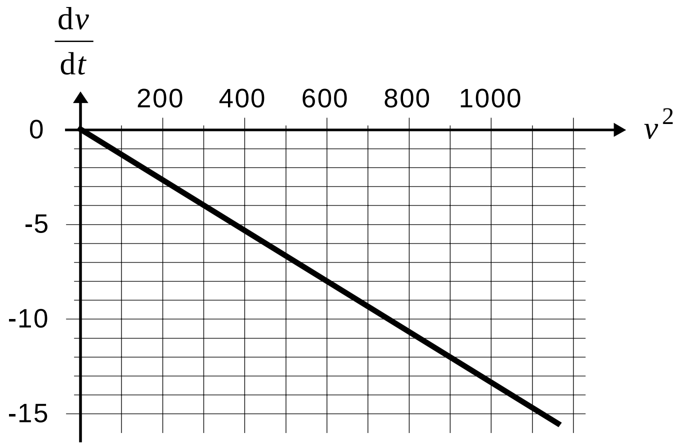
10. Two of the faces of a cuboid have areas of 10.0 cm^2 and 20.0 cm^2 respectively.

Use differentiation to find all the possible values of the total length of the edges. (5p)



11. A fast boat weighing 1200 kg cruises at 30 m/s in calm water when the motor suddenly stops and the boat is slowed down by the water. Let v m/s be the speed of the boat t seconds after the motor stops.

As it is shown in the graph below, the rate of change of the speed $\frac{dv}{dt}$ in m/s^2 is a function which depends on the square of the speed, v^2.

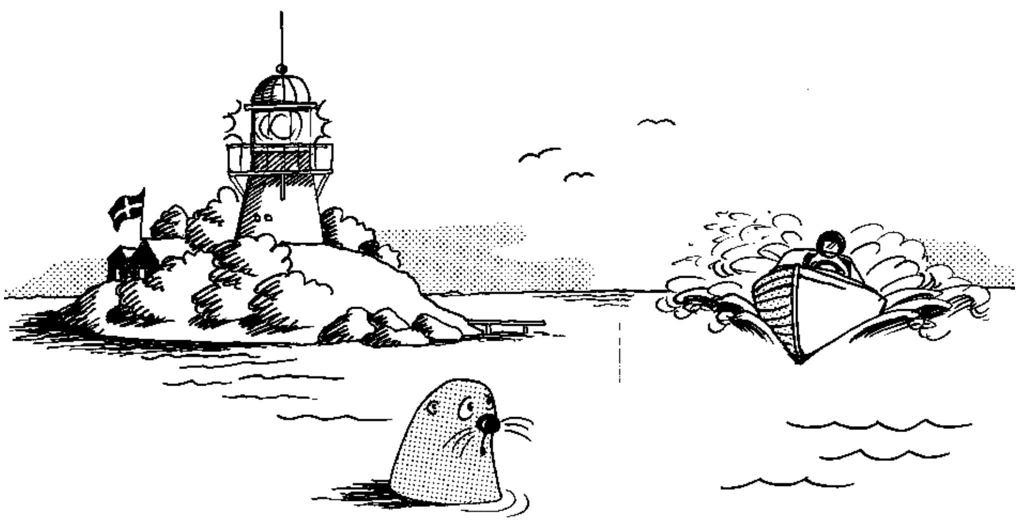


a) Use the graph to find a differential equation which decsribes the decrease of speed after the motor failure. (2p)

b) One solution to the differential equation is $v = \dfrac{1200}{16t + C}$, where C is a constant. Use this expression to find the speed of the boat 2 seconds after the motor stops. (2p)

c) How far a distance does the boat travel during the first 2 seconds after the motor drops dead? (2p)

d) Starting from the differential equation, find the speed of the boat 2 seconds after the motor stops by using a numerical method. Compare your result with what you obtained in b), and explain why you cannot expect them to be in accordance. (4p)



12. The differentiable function $y = f(x)$ is increasing $0 \le f'(x) \le 0.5$ when $x \ge 0$.
Three values of the function are given in the table.

x	y
30	5.9
50	8.1
90	11.0

Find the smallest possible number b which satisfies

$$\int_0^{130} f(x)\mathrm{d}x < b$$

(6p)