Directions

Test period	April 24 – June 3 1998.
Test time	240 minutes.
Resources	Part I: A collection of formulas Part II: A non-symbol manipulating graphics calculator and a collection of formulas.
Test material	This test paper should be handed in with your solutions. The solutions to part I should be handed in before you may have access to your calculator. Therefore you must use a separate sheet of paper for your solutions to part I. Note that you may begin your work on part II without your calculator. Write your name and the name of your education programme/adult education on all the sheets of paper you hand in.
The test	The test consists of 18 problems. Most of the problems are of the *long-answer type*. In these problems it is not enough to give short answers, they require • that you write down what you do • that you explain your train of thought • that you, where necessary, draw figures • that you show how you have used your resources when you have solved problems numerically/graphically For some problems (where it says *Only an answer is required*) you only need to give the answer. Try all of the problems. It can be relatively easy, even towards the end of the test, to earn some points for a partial solution or presentation.
The score levels	The teacher responsible will inform you about the scores required for ”Passed” and ”Passed with Distinction”. The maximum score is 58 points.

PART I

This part contains 11 problems, which should be solved without a calculator. You must do your solutions to this part on a separate sheet of paper that will be handed in before you may have access to your calculator. Please note that the work on Part II can be started without the calculator.

1. Solve the equation $z^2 = 2z - 5$ (2p)

2. Solve the differential equation $y' + 3y = 0$, $y(0) = 5$ (2p)

3. Express $\dfrac{1+3\mathrm{i}}{3+\mathrm{i}}$ in the form $a + b\mathrm{i}$ where a and b are real numbers. (2p)

4. Solve the differential equation $y'' - 12y' + 32y = 0$ (2p)

5. Solve the equation $z + \mathrm{i}z = \mathrm{i}$ (2p)

6. Give an example of a quadratic equation that

a) does not have any real roots *Only an answer is required* (1p)

b) has two real roots *Only an answer is required* (1p)

7. For the complex number z it holds that $|z| = 3$
Represent all possible positions for z and $\bar{z}$ in the complex plane and describe in words the possible positions. (3p)

8. a) By solutions to equations we usually mean numbers that satisfy certain conditions. For example, the number 3 is a solution to the cubic equation $x^3 - 27 = 0$.
What do we mean by solutions to differential equations? (1p)

b) Investigate if there is a solution to the differential equation $3y' - 10y = 0$ that satisfies the two conditions $y(0) = 1$ **and** $y'(0) = 1$ (2p)

9. Show that $\dfrac{6e^{-\mathrm{i}\pi/3}}{3e^{\mathrm{i}\pi/3}} = -1 - \mathrm{i}\sqrt{3}$ (3p)

10. In the expression $y = kx^2$, y and x are functions of time t and k is a constant, $k = \dfrac{1}{20}$

Find x at the moment when $\dfrac{\mathrm{d}y}{\mathrm{d}t} = 3$ and $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 8$ (3p)

11. a) Choose two complex numbers z and w such that $\operatorname{Im} z \neq 0$ and $\operatorname{Im} w \neq 0$.
Show that for your chosen complex numbers it holds that $\overline{zw} = \overline{z} \cdot \overline{w}$. (2p)

b) Show that for any complex numbers it holds that $\overline{zw} = \overline{z} \cdot \overline{w}$. (2p)

PART II

This part contains 7 problems, which are intended to be solved with the aid of a graphics calculator (non-symbol manipulating). Note that you may begin your work on part II without your calculator.

12. Solve the differential equation $y'' + 9y = 0$ when $y(0) = 0$ and $y'(0) = 3$. (3p)

13. A company that produces collector's pictures is concerned about the fact that the sale of its product has been on a constant level during a few months. Therefore, the company carries out an advertising campaign which results in an increased sale. After the advertising campaign, the rate of change in the number of sold items at each moment is proportional to the square root of the number of sold items at that moment.

Express the rate of change of the number of sold items in a differential equation

a) before the advertising campaign *Only an answer is required* (1p)

b) after the advertising campaign *Only an answer is required* (2p)

14. The probability that an electronic component will break between *a* hours and *b* hours is given by the integral $\int_a^b f(x)dx$

The function $f(x)$ is the so-called frequency function, which in this case is

$$f(x) = \begin{cases} 0.001e^{-0.001x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Calculate the probability that such a component will break in the interval between 0 and 500 hours. (2p)

15. From a point P on the curve $y = 3x^2 - 2\ln 2x + 2$, lines are drawn perpendicular to the *x*-axis and *y*-axis respectively. Together with the axes, these lines bound a rectangle.

a) Find an expression for the perimeter of the rectangle as a function of the *x*-coordinate at the point P. (2p)

b) Find the minimum value of the perimeter by using the derivative. (3p)

16. During a chemical experiment, the quantity of a certain substance decreases. At different points of time it is analysed how many per cent of the substance that remains. The following results are recorded:

time in minutes	48	76	124	204	238	289
percentage left	82.7	74.3	61.1	44.4	38.7	31.1

If the remaining quantity of the substance is denoted *y* % and the time (in minutes) is denoted *t*, the experiment can be described by the differential equation $\frac{dy}{dt} = ky$

Find, as exact as you can, a value of the constant of proportionality *k* (3p)

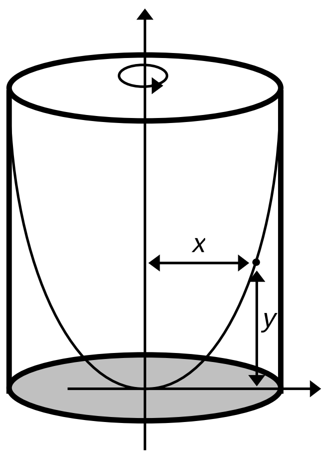
17. Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{x^2 - 4}$, $y(0) = 1$

a) Give an account for a numerical method of finding a value for $y(1)$ (2p)

b) Find $y(1)$ using a numerical method with at least four steps. *Only an answer is required* (1p)

c) Explain why this numerical method cannot be used when calculating $y(3)$ (2p)

18. A cylindrical container with radius 12 cm is filled with water. The container is rotated and, as long as the speed of rotation increases, water flows over the brim of the container. At a certain speed of rotation, the water level at the centre of the tank becomes zero, see figure. Under these conditions, it holds that $y' = 0.20x$, where y' is the slope of the surface of water on the distance *x* cm from the axis of rotation.



a) Find *y* as a function of *x* (2p)

b) Calculate how much water that has flowed out since the rotation started. (4p)

c) The speed of the rotation is increased so that a circular area with radius 3.0 cm is drained in the middle of the cylinder. In the expression $y' = kx,\ \ x \geq 3$, *k* then achieves a new value. The remaining water in the cylinder will still reach the brim. Write down an expression for the volume of the remaining water. (You need not calculate the volume.) (3p)