Directions

Test time	240 minutes.
Resources	Part I: A collection of formulas. Part II: A non-symbol manipulating graphics calculator and a collection of formulas.
Test material	This test paper should be handed in with your solutions. The solutions to part I should be handed in before you may have access to your calculator. Therefore you must use a separate sheet of paper for your solutions to part I. Note that you may begin your work on part II without your calculator. Write your name, the name of your education programme / adult education, and your date of birth on all the sheets of paper you hand in.
The test	The test consists of 15 problems. Most of the problems are of the *long-answer type*. In these problems it is not enough to give short answers, they require • that you write down what you do • that you explain your train of thought • that you, where necessary, draw figures • that you show how you have used your resources when you have solved problems numerically/graphically For some problems (where it says *Only an answer is required*) you only need to give the answer. NOTE! The decimal point is placed at the bottom of the line i.e. 3.5 = 3½ Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for a partial solution or presentation.
The score levels	The teacher responsible will inform you about the scores required for ”Passed” and ”Passed with Distinction”. The maximum score is 49 points.

PART I

This part consists of 9 problems and is intended to be carried through without the use of a calculator.
Your solutions to this part should be presented on separate sheets of paper that must be handed in before you get access to a calculator.
Please note that the work with Part II can be started without the calculator.

1. Express the complex number $\dfrac{7-3\mathrm{i}}{2-\mathrm{i}}$ in the form $x+y\mathrm{i}$. (2p)

2. Solve the equation

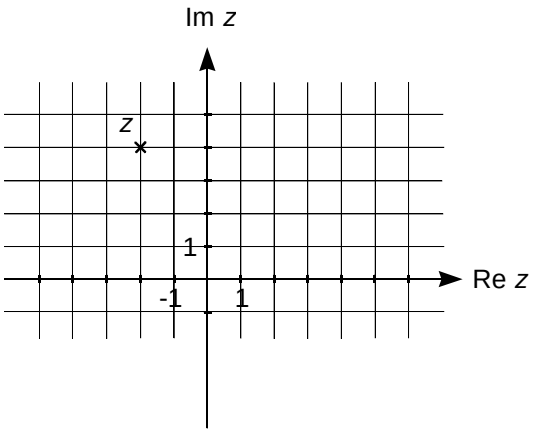
a) $z^2-2z+5=0$ (2p)

b) $3(z-3)^2+48=0$ (2p)

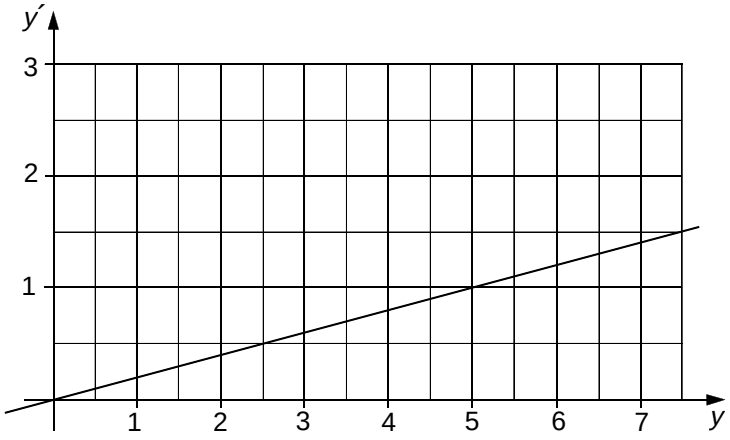
3. a) Find the argument and modulus of the complex number $\sqrt{3}+\mathrm{i}$
Only an answer is required (2p)

b) Find the argument of $\left(\sqrt{3}+\mathrm{i}\right)^6$ *Only an answer is required* (1p)

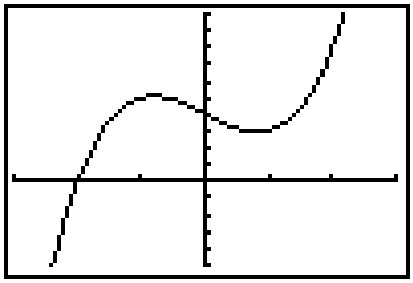
4. The complex number z is shown in the complex plane.
Find $z\bar{z}$ (2p)



5. The figure below shows how the function y and its derivative y' are related. Express this relation with a differential equation. Then, determine the function $y(x)$ if $y(0) = 2$ (3p)

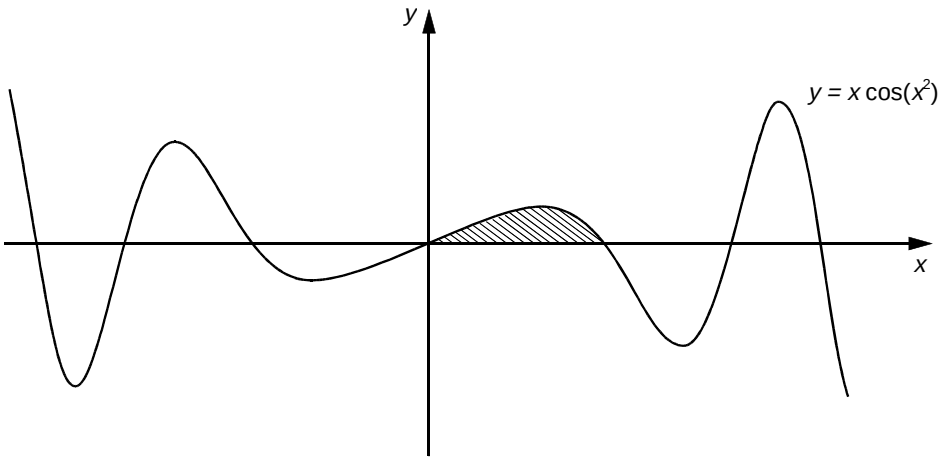


6. The figure below shows the graph to the function $y = x^3 - 2x + 4$
Find all roots to the equation $x^3 - 2x + 4 = 0$ (3p)



7. a) Find $f'(x)$ if $f(x) = \sin(x^2)$ *Only an answer is required* (1p)

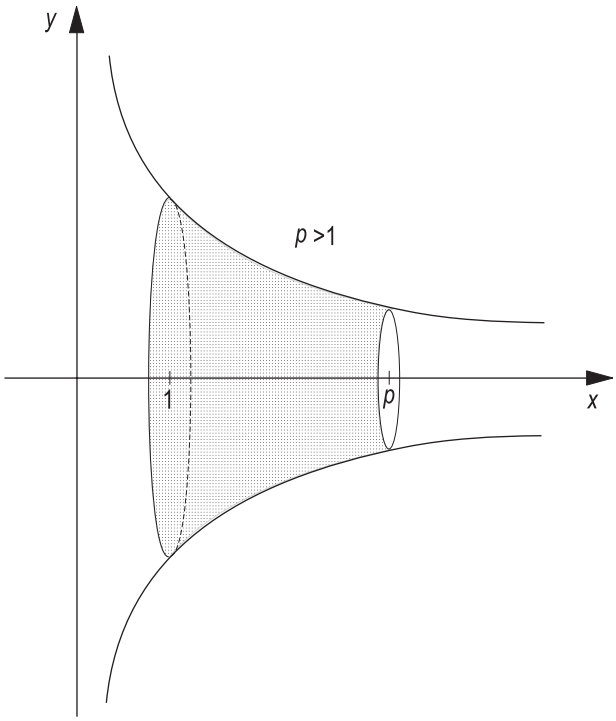
b) Use the result in a) and calculate the area of the shaded region. (3p)



8. The curve $y = \frac{1}{x}$ is rotated about the *x*-axis (see the figure below).

a) Calculate the shaded volume when $p = 2$ (3p)

b) Investigate wether there is a value of *p* that gives a volume twice as large as in a). (2p)



9. Find the values of *k* where $e^{ki\pi} + 1 = 0$? (2p)

PART II

This part consists of 6 problems and a calculator can be used (graphic but not with symbolic computation). Please note that the work with Part II can be started without the access to a calculator**.**

10.

In the summer of 1845, parts of Western Europe suffered from the potato plague. In Ireland the situation was serious, partly because the land was overpopulated, partly because more than half of the population depended completely on potatoes as provisions. The plague returned in 1846 and 1847 and many Irishmen	*starved to death, died from diseases or emigrated to the US and Canada.* *Afterwards, "the Great Famine" 1846-1848 appears as an important event in the Irish history: the emigration accelerated and the population decreased.* *(Source: Nationalencyklopedin)*

If y is the number of inhabitants in Ireland t years after 1850, the following statement was true during a certain period:

$\frac{\mathrm{d}y}{\mathrm{d}t} = -0.012y, \qquad y(0) = 6.5\times10^6$

In words, describe what these expressions mean in relation to the population of Ireland during this period. (2p)

11. Solve the differential equation $y'' + 6y' + 8y = 0$ when $y(0) = 1$ and $y'(0) = 0$ (3p)

12. The average temperature during a certain period from a to b can be calculated from the formula $\frac{1}{b-a}\int_a^b y\,\mathrm{d}x$ when y ºC describes temperature as a function of time.

At a certain location, the temperature was registrated during twenty-four hours. It was found that the temperature could be described by the function $y = 3\sin(0.3x - 3) + 7.7$ where x is the number of hours after midnight.

Calculate the average temperature for the twenty-four hour period.

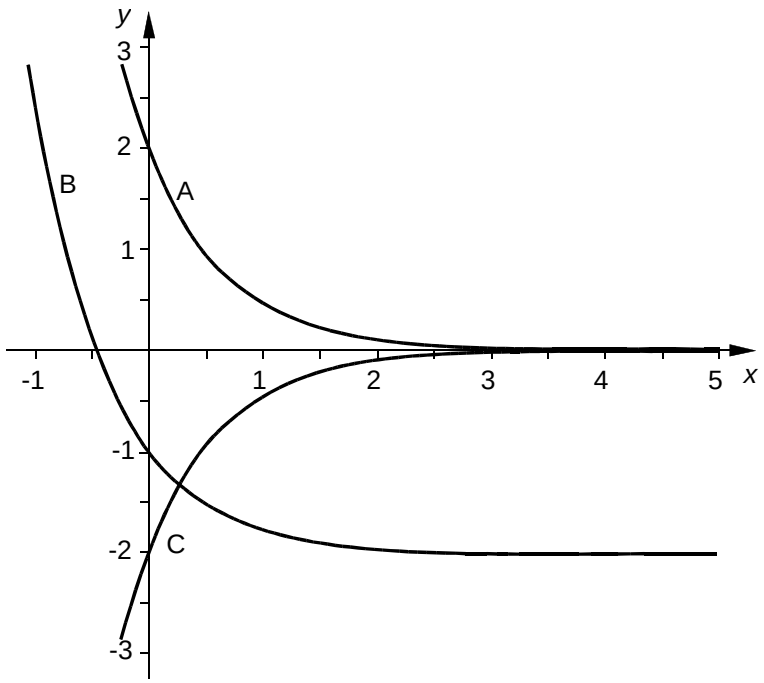
. (3p)

13. This exercise deals with solutions to the differential equation $2y' + 3y = 0$

a) Find y when $y'(0) = -6$ (2p)

b) In the figure below, curve A represents another solution to the differential equation $2y' + 3y = 0$. Find this solution. *Only an answer is required* (1p)

c) Can the curves B and C in the figure be solutions to the differential equation $2y' + 3y = 0$? Justify your answer. (2p)



14. Kalle was involved in an accident at his work where he happened to inhale toxic fumes from a chemical substance.

It was a long time before Kalle went to the hospital, and a blood test was not taken until 20 hours after the accident. The analysis showed that his blood contained 0.00372 mg/ml of the poison he had inhaled.

After another 8 hours, a new blood test was taken, and by then the concentration of poison in his blood had decreased to 0.00219 mg/ml.

Let us assume that the rate of change of the concentration of poison is proportional to the concentration, and let *y* mg/ml be the concentration of poison in his blood *t* hours after the first blood test.

The doctor wants to give him medical treatment if the concentration of poison at any time has been higher than 0.017 mg/ml. According to the model, is there any risk that the concentration of poison in Kalle's blood has been that high? (4p)

15. When $a = 8$ the equation $z^2 - 6z + a = 0$ has roots $z_1 = 2$ and $z_2 = 4$. Investigate for all real values of a where in the complex plane the roots to the equation can be found. (4p)