Formulas for mathematics 3

Algebra

Rules

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Quadratic equations

$$x^2 + px + q = 0 \qquad x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$ax^2 + bx + c = 0 \qquad x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Arithmetic

Prefixes

T	G	M	k	h	d	c	m	μ	n	p
tera	giga	mega	kilo	hecto	deci	centi	milli	micro	nano	pico
10^{12}	10^{9}	10^{6}	10^{3}	10^{2}	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}

Powers

$$a^x a^y = a^{x+y} \qquad \frac{a^x}{a^y} = a^{x-y} \qquad (a^x)^y = a^{xy} \qquad a^{-x} = \frac{1}{a^x}$$

$$a^x b^x = (ab)^x \qquad \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^0 = 1$$

Geometric series

$$a + ak + ak^2 + \ldots + ak^{n-1} = \frac{a(k^n - 1)}{k - 1} \quad \text{where } k \neq 1$$

Logarithms

$$y = 10^x \Leftrightarrow x = \lg y \qquad y = \mathrm{e}^x \Leftrightarrow x = \ln y$$

$$\lg x + \lg y = \lg xy \qquad \lg x - \lg y = \lg \frac{x}{y} \qquad \lg x^p = p \cdot \lg x$$

Absolute value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Functions and relations

Linear function

$y = kx + m \qquad k = \dfrac{y_2 - y_1}{x_2 - x_1}$

$ax + by + c = 0$, where a and b are not both zero

Quadratic function

$y = ax^2 + bx + c \qquad a \neq 0$

Power function

$y = C \cdot x^a$

Exponential function

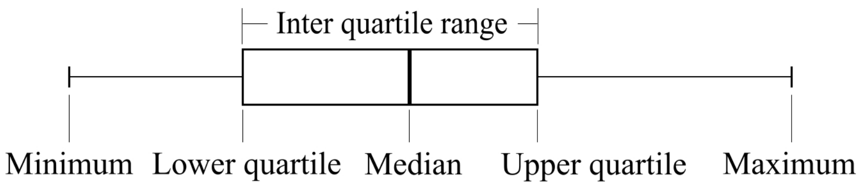
$y = C \cdot a^x \qquad a > 0$ och $a \neq 1$

Statistics and probability

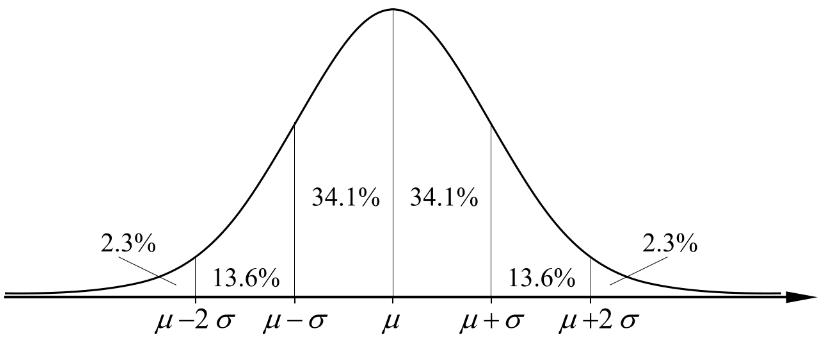
Standard deviation of a sample

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n-1}}$$

Box plot



Normal distribution



Differential and integral calculus

Definition of the derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Derivative

Function	Derivative
x^n where n is a real number	nx^{n-1}
a^x $(a > 0)$	$a^x \ln a$
e^x	e^x
e^{kx}	$k \cdot \mathrm{e}^{kx}$
$\frac{1}{x}$	$-\frac{1}{x^2}$
$k \cdot f(x)$	$k \cdot f'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$

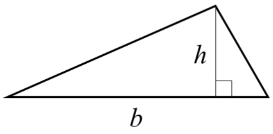
Antiderivatives

Function	Antiderivatives
k	$kx + C$
x^n $(n \neq -1)$	$\frac{x^{n+1}}{n+1} + C$
e^x	$\mathrm{e}^x + C$
e^{kx}	$\frac{\mathrm{e}^{kx}}{k} + C$
a^x $(a > 0,\ a \neq 1)$	$\frac{a^x}{\ln a} + C$

Geometry

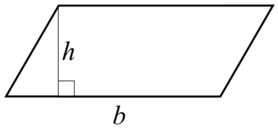
Triangle

$$A = \frac{bh}{2}$$



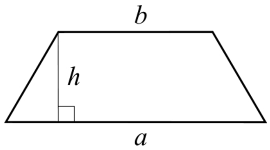
Parallelogram

$$A = bh$$



Trapezium

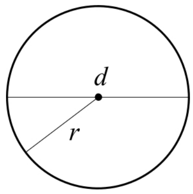
$$A = \frac{h(a+b)}{2}$$



Circle

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

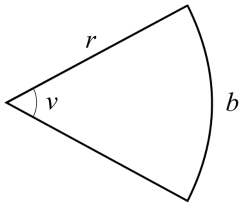
$$O = 2\pi r = \pi d$$



Circle sector

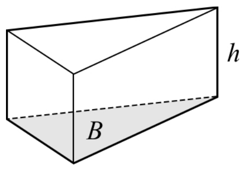
$$b = \frac{v}{360°} \cdot 2\pi r$$

$$A = \frac{v}{360°} \cdot \pi r^2 = \frac{br}{2}$$



Prism

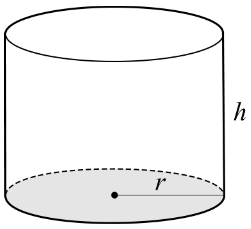
$$V = Bh$$



Cylinder

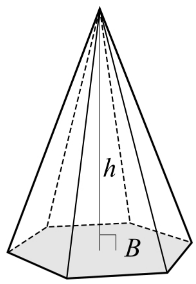
$$V = \pi r^2 h$$

Lateral surface area
$$A = 2\pi rh$$



Pyramid

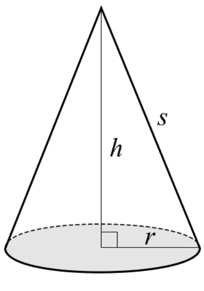
$$V = \frac{Bh}{3}$$



Cone

$$V = \frac{\pi r^2 h}{3}$$

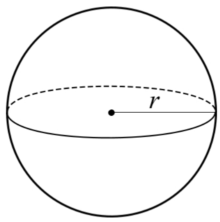
Lateral surface area
$$A = \pi rs$$



Sphere

$$V = \frac{4\pi r^3}{3}$$

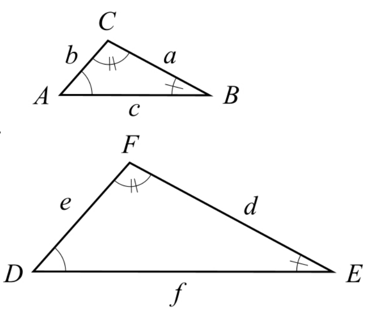
$$A = 4\pi r^2$$



Similarity

The triangles ABC and DEF are similar.

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$



Scale

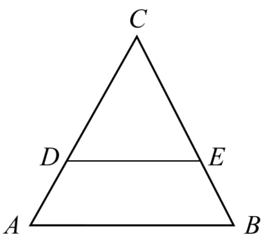
Area scale factor = (Length scale factor)^2
Volume scale factor = (Length scale factor)^3

Triangle with a transversal line

If DE is parallel to AB then

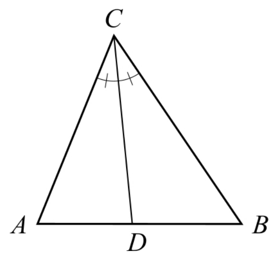
$$\frac{DE}{AB} = \frac{CD}{AC} = \frac{CE}{BC} \text{ and}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$



Angle bisector theorem

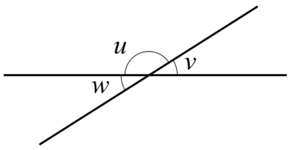
$$\frac{AD}{BD} = \frac{AC}{BC}$$



Angles

$u + v = 180°$ Supplementary angles

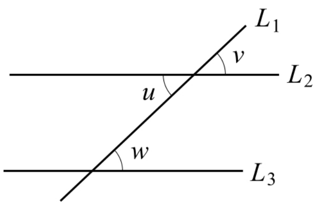
$w = v$ Vertical angles



L_1 cuts two parallel lines L_2 and L_3

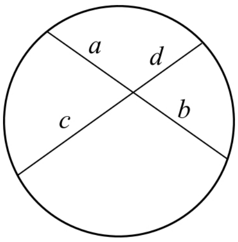
$v = w$ Corresponding angles

$u = w$ Alternate angles



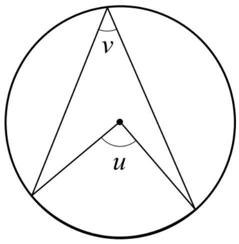
Chord theorem

$ab = cd$



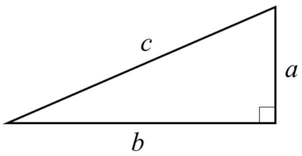
Angles subtended by the same arc

$u = 2v$



Pythagoras' theorem

$a^2 + b^2 = c^2$



Distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint formula

$$x_m = \frac{x_1 + x_2}{2} \text{ and } y_m = \frac{y_1 + y_2}{2}$$

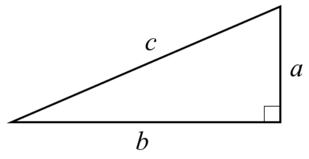
Trigonometry

Definitions

$$\sin v = \frac{a}{c}$$

$$\cos v = \frac{b}{c}$$

$$\tan v = \frac{a}{b}$$

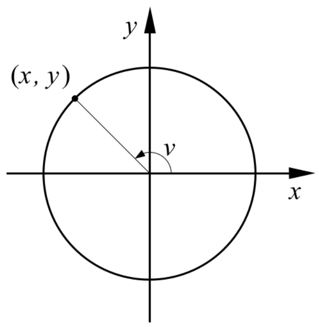


Unit circle

$$\sin v = y$$

$$\cos v = x$$

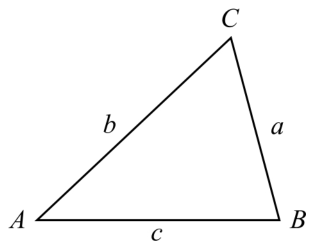
$$\tan v = \frac{y}{x}$$



Sine rule $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine rule $$a^2 = b^2 + c^2 - 2bc\cos A$$

Area $$T = \frac{ab\sin C}{2}$$



Circle equation $$(x-a)^2 + (y-b)^2 = r^2$$

Exact values

Angle v	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0