

Formulas for mathematics 3

Algebra

Rules

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Quadratic equations

$$x^2 + px + q = 0$$

$$ax^2 + bx + c = 0$$

$$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Arithmetic

Prefixes

| T | G | M | k | h | d | c | m | μ | n | p |
|-----------|--------|--------|--------|--------|-----------|-----------|-----------|-----------|-----------|------------|
| tera | giga | mega | kilo | hecto | deci | centi | milli | micro | nano | pico |
| 10^{12} | 10^9 | 10^6 | 10^3 | 10^2 | 10^{-1} | 10^{-2} | 10^{-3} | 10^{-6} | 10^{-9} | 10^{-12} |

Powers

$$a^x a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^{-x} = \frac{1}{a^x}$$

$$a^x b^x = (ab)^x$$

$$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^0 = 1$$

Geometric series

$$a + ak + ak^2 + \dots + ak^{n-1} = \frac{a(k^n - 1)}{k - 1} \quad \text{where } k \neq 1$$

Logarithms

$$y = 10^x \Leftrightarrow x = \lg y$$

$$y = e^x \Leftrightarrow x = \ln y$$

$$\lg x + \lg y = \lg xy$$

$$\lg x - \lg y = \lg \frac{x}{y}$$

$$\lg x^p = p \cdot \lg x$$

Absolute value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Functions and relations

Linear function

$$y = kx + m \quad k = \frac{y_2 - y_1}{x_2 - x_1}$$

$k_1 \cdot k_2 = -1$, condition for perpendicular lines

$ax + by + c = 0$, where a and b are not both zero

Quadratic functions

$$y = ax^2 + bx + c \quad a \neq 0$$

Power functions

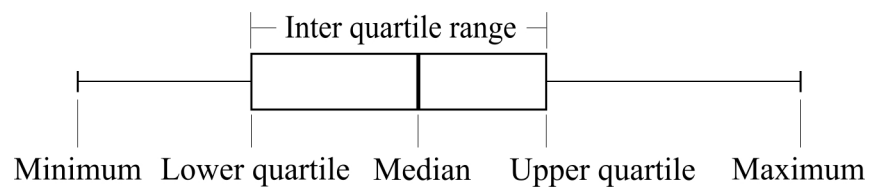
$$y = C \cdot x^a$$

Exponential functions

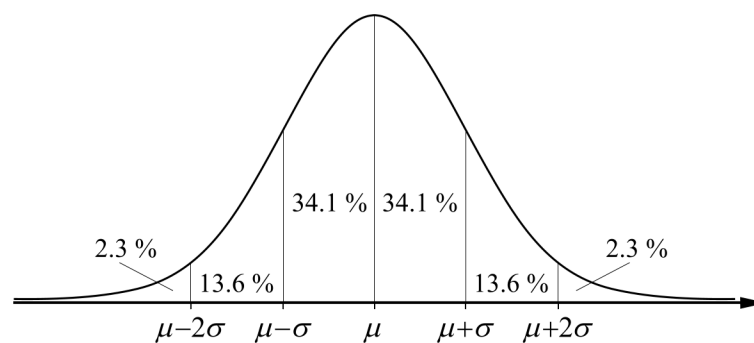
$$y = C \cdot a^x \quad a > 0 \text{ and } a \neq 1$$

Statistics and probability

Box plot



Normal distribution



Differential and integral calculus

Definition of the derivative

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Derivatives

| Function | Derivative |
|----------------------------------|------------------|
| x^n where n is a real number | nx^{n-1} |
| $\frac{1}{x}$ | $-\frac{1}{x^2}$ |
| a^x ($a > 0$) | $a^x \ln a$ |
| e^x | e^x |
| e^{kx} | $k \cdot e^{kx}$ |
| $k \cdot f(x)$ | $k \cdot f'(x)$ |
| $f(x) + g(x)$ | $f'(x) + g'(x)$ |

Fundamental theorem of calculus

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a) \quad \text{where } F'(x) = f(x)$$

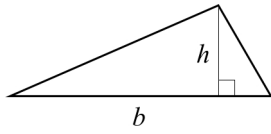
Antiderivatives

| Function | Antiderivatives |
|-----------------------------|---------------------------|
| k | $kx + C$ |
| x^n ($n \neq -1$) | $\frac{x^{n+1}}{n+1} + C$ |
| a^x ($a > 0, a \neq 1$) | $\frac{a^x}{\ln a} + C$ |
| e^x | $e^x + C$ |
| e^{kx} | $\frac{e^{kx}}{k} + C$ |

Geometry

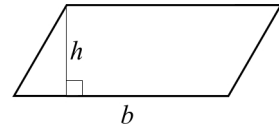
Triangle

$$A = \frac{bh}{2}$$



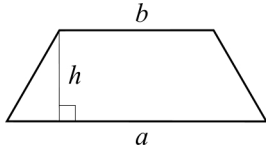
Parallelogram

$$A = bh$$



Trapezium

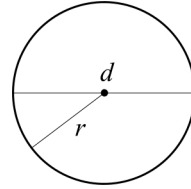
$$A = \frac{h(a+b)}{2}$$



Circle

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

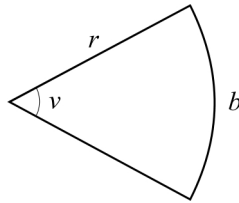
$$O = 2\pi r = \pi d$$



Circle sector

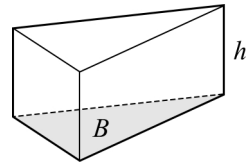
$$b = \frac{v}{360^\circ} \cdot 2\pi r$$

$$A = \frac{v}{360^\circ} \cdot \pi r^2 = \frac{br}{2}$$



Prism

$$V = Bh$$

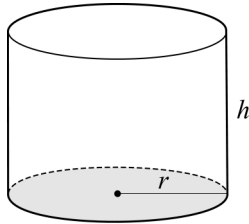


Cylinder

$$V = \pi r^2 h$$

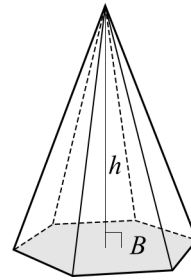
Lateral surface area

$$A = 2\pi r h$$



Pyramid

$$V = \frac{Bh}{3}$$

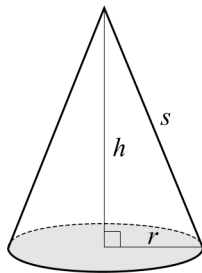


Cone

$$V = \frac{\pi r^2 h}{3}$$

Lateral surface area

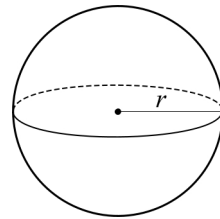
$$A = \pi r s$$



Sphere

$$V = \frac{4\pi r^3}{3}$$

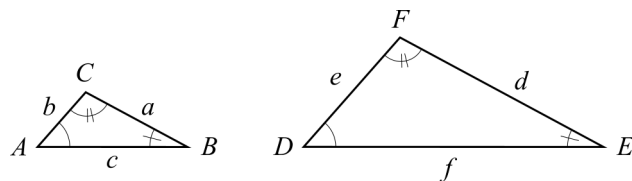
$$A = 4\pi r^2$$



Similarity

The triangles ABC and DEF are

similar if $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

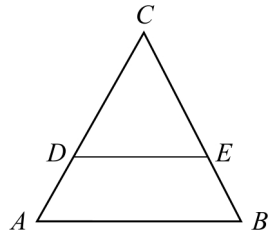


Scale Area scale factor = (Length scale factor)² Volume scale factor = (Length scale factor)³

Triangle with a transversal line

$$\frac{DE}{AB} = \frac{CD}{AC} = \frac{CE}{BC} \text{ and}$$

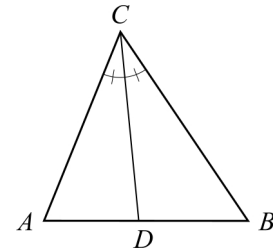
$$\frac{CD}{AD} = \frac{CE}{BE}$$



DE is parallel to AB

Angle bisector theorem

$$\frac{AD}{BD} = \frac{AC}{BC}$$



Angles

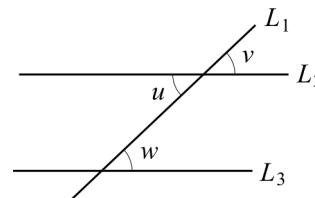
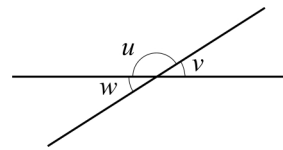
$u + v = 180^\circ$ Supplementary angles

$w = v$ Vertical angles

L_1 intersects two parallel lines L_2 and L_3

$v = w$ Corresponding angles

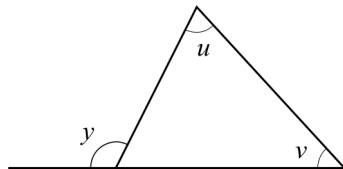
$u = w$ Alternate angles



The sum S of all angles of a n -polygon: $S = (n - 2) \cdot 180^\circ$

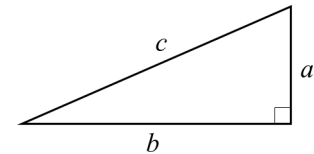
Exterior angle theorem

$$y = u + v$$



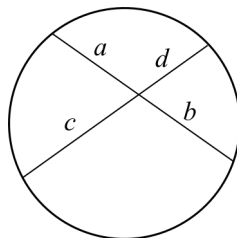
Pythagoras' theorem

$$a^2 + b^2 = c^2$$



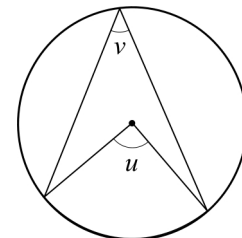
Chord theorem

$$ab = cd$$



Angles subtended by the same arc

$$u = 2v$$



Distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

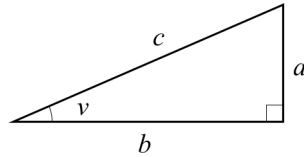
Midpoint formula

$$x_m = \frac{x_1 + x_2}{2} \text{ and } y_m = \frac{y_1 + y_2}{2}$$

Trigonometry

Definitions

Right-angled triangle

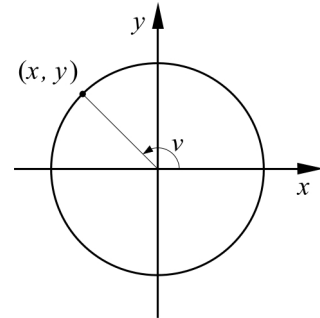


$$\sin v = \frac{a}{c}$$

$$\cos v = \frac{b}{c}$$

$$\tan v = \frac{a}{b}$$

Unit circle



$$\sin v = y$$

$$\cos v = x$$

$$\tan v = \frac{y}{x}$$

Law of sines

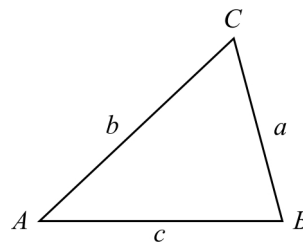
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area formula

$$T = \frac{ab \sin C}{2}$$



Values of trigonometric functions

| Angle v | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
|-----------|-----------|----------------------|----------------------|----------------------|------------|----------------------|-----------------------|-----------------------|-------------|
| $\sin v$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\cos v$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{\sqrt{3}}{2}$ | -1 |
| $\tan v$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | Not def. | $-\sqrt{3}$ | -1 | $-\frac{1}{\sqrt{3}}$ | 0 |