

Formulas for mathematics 4

Algebra

Rules

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Quadratic equations

$$x^2 + px + q = 0$$

$$ax^2 + bx + c = 0$$

$$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Arithmetic

Prefixes

| T | G | M | k | h | d | c | m | μ | n | p |
|-----------|--------|--------|--------|--------|-----------|-----------|-----------|-----------|-----------|------------|
| tera | giga | mega | kilo | hecto | deci | centi | milli | micro | nano | pico |
| 10^{12} | 10^9 | 10^6 | 10^3 | 10^2 | 10^{-1} | 10^{-2} | 10^{-3} | 10^{-6} | 10^{-9} | 10^{-12} |

Powers

$$a^x a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^{-x} = \frac{1}{a^x}$$

$$a^x b^x = (ab)^x$$

$$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^0 = 1$$

Geometric series

$$a + ak + ak^2 + \dots + ak^{n-1} = \frac{a(k^n - 1)}{k - 1} \quad \text{where } k \neq 1$$

Logarithms

$$y = 10^x \Leftrightarrow x = \lg y$$

$$y = e^x \Leftrightarrow x = \ln y$$

$$\lg x + \lg y = \lg xy$$

$$\lg x - \lg y = \lg \frac{x}{y}$$

$$\lg x^p = p \cdot \lg x$$

Absolute value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Functions and relations

Linear function

$$y = kx + m \quad k = \frac{y_2 - y_1}{x_2 - x_1}$$

$ax + by + c = 0$, where a and b are not both zero

Quadratic function

$$y = ax^2 + bx + c \quad a \neq 0$$

Power function

$$y = C \cdot x^a$$

Exponential function

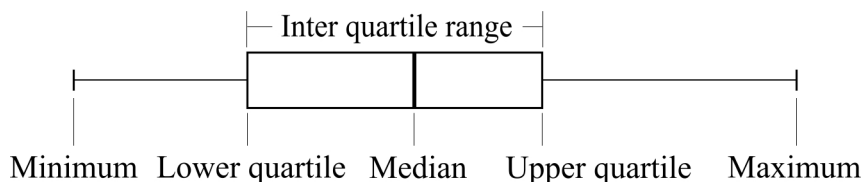
$$y = C \cdot a^x \quad a > 0 \text{ och } a \neq 1$$

Statistics and probability

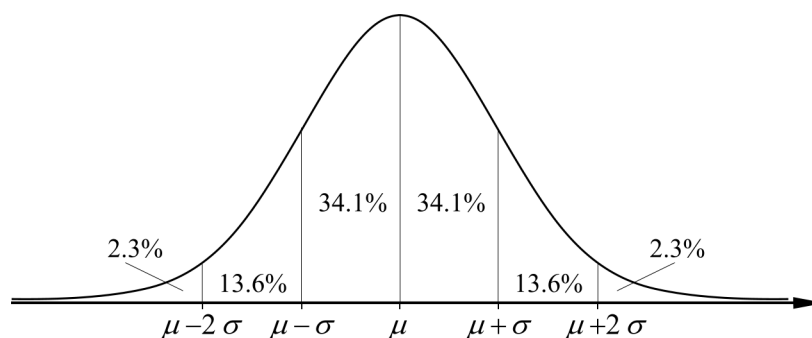
Standard deviation of a sample

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}}$$

Box plot



Normal distribution



Density function of the normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Differential and integral calculus

Definition of the derivative

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Derivative

| Function | Derivative |
|---------------------------------------|--|
| x^n where n is a real number | nx^{n-1} |
| a^x ($a > 0$) | $a^x \ln a$ |
| $\ln x$ ($x > 0$) | $\frac{1}{x}$ |
| e^x | e^x |
| e^{kx} | $k \cdot e^{kx}$ |
| $\frac{1}{x}$ | $-\frac{1}{x^2}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $1 + \tan^2 x = \frac{1}{\cos^2 x}$ |
| $k \cdot f(x)$ | $k \cdot f'(x)$ |
| $f(x) + g(x)$ | $f'(x) + g'(x)$ |
| $f(x) \cdot g(x)$ | $f'(x) \cdot g(x) + f(x) \cdot g'(x)$ |
| $\frac{f(x)}{g(x)}$ ($g(x) \neq 0$) | $\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$ |

Chain rule

If $y = f(z)$ and $z = g(x)$ are two differentiable functions then it holds for $y = f(g(x))$ that

$$y' = f'(g(x)) \cdot g'(x) \text{ or } \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

Antiderivatives

| Function | Antiderivatives |
|-------------------------------|---------------------------|
| k | $kx + C$ |
| $x^n \quad (n \neq -1)$ | $\frac{x^{n+1}}{n+1} + C$ |
| $\frac{1}{x}$ | $\ln x + C \quad (x > 0)$ |
| e^x | $e^x + C$ |
| e^{kx} | $\frac{e^{kx}}{k} + C$ |
| $a^x \quad (a > 0, a \neq 1)$ | $\frac{a^x}{\ln a} + C$ |
| $\sin x$ | $-\cos x + C$ |
| $\cos x$ | $\sin x + C$ |

Complex numbers**Representation**

$$z = x + iy = re^{iv} = r(\cos v + i \sin v) \text{ where } i^2 = -1$$

Argument

$$\arg z = v \quad \tan v = \frac{y}{x}$$

Absolute value

$$|z| = r = \sqrt{x^2 + y^2}$$

Conjugate

$$\text{If } z = x + iy \text{ then } \bar{z} = x - iy$$

Rules

$$z_1 z_2 = r_1 r_2 (\cos(v_1 + v_2) + i \sin(v_1 + v_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos(v_1 - v_2) + i \sin(v_1 - v_2))$$

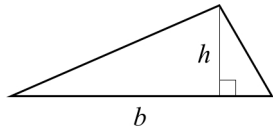
de Moivre's formula

$$z^n = (r(\cos v + i \sin v))^n = r^n (\cos nv + i \sin nv)$$

Geometry

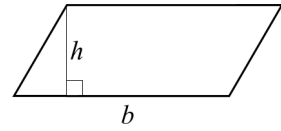
Triangle

$$A = \frac{bh}{2}$$



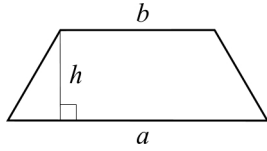
Parallelogram

$$A = bh$$



Trapezium

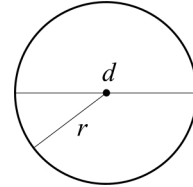
$$A = \frac{h(a+b)}{2}$$



Circle

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

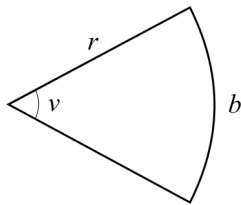
$$O = 2\pi r = \pi d$$



Circle sector

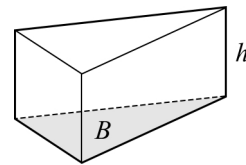
$$b = \frac{v}{360^\circ} \cdot 2\pi r$$

$$A = \frac{v}{360^\circ} \cdot \pi r^2 = \frac{br}{2}$$



Prism

$$V = Bh$$

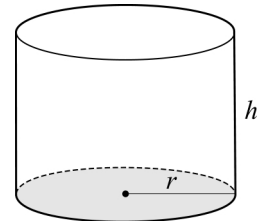


Cylinder

$$V = \pi r^2 h$$

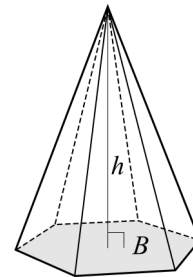
Lateral surface area

$$A = 2\pi r h$$



Pyramid

$$V = \frac{Bh}{3}$$

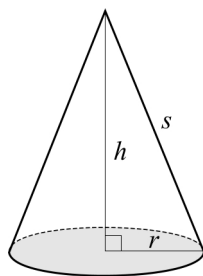


Cone

$$V = \frac{\pi r^2 h}{3}$$

Lateral surface area

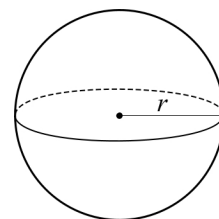
$$A = \pi r s$$



Sphere

$$V = \frac{4\pi r^3}{3}$$

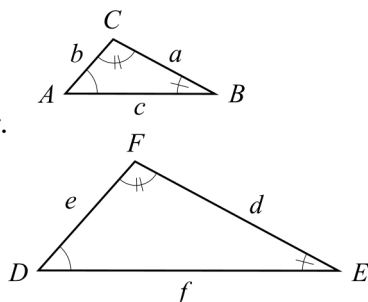
$$A = 4\pi r^2$$



Similarity

The triangles ABC and DEF are similar.

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$



Scale

Area scale factor = (Length scale factor)²

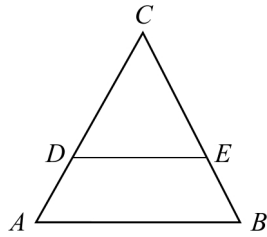
Volume scale factor = (Length scale factor)³

Triangle with a transversal line

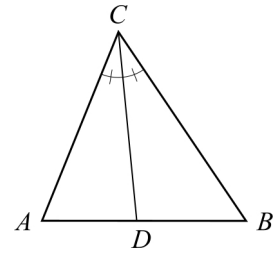
If DE is parallel to AB then

$$\frac{DE}{AB} = \frac{CD}{AC} = \frac{CE}{BC} \text{ and}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$

**Angle bisector theorem**

$$\frac{AD}{BD} = \frac{AC}{BC}$$

**Angles**

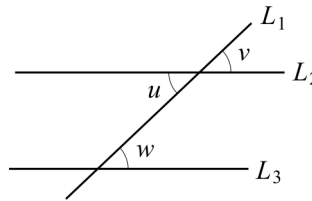
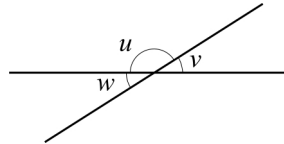
$$u + v = 180^\circ \quad \text{Supplementary angles}$$

$$w = v \quad \text{Vertical angles}$$

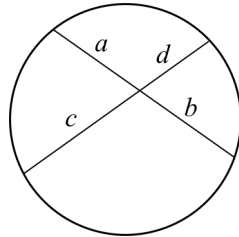
L_1 cuts two parallel lines L_2 and L_3

$$v = w \quad \text{Corresponding angles}$$

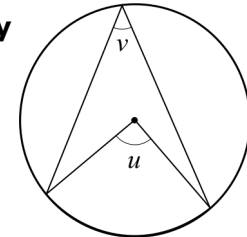
$$u = w \quad \text{Alternate angles}$$

**Chord theorem**

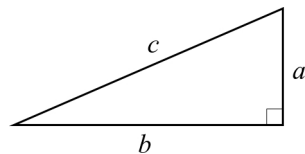
$$ab = cd$$

**Angles subtended by the same arc**

$$u = 2v$$

**Pythagoras' theorem**

$$a^2 + b^2 = c^2$$

**Distance formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint formula

$$x_m = \frac{x_1 + x_2}{2} \text{ and } y_m = \frac{y_1 + y_2}{2}$$

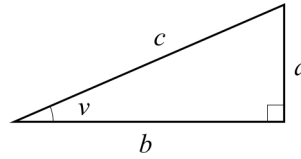
Trigonometry

Definitions

$$\sin v = \frac{a}{c}$$

$$\cos v = \frac{b}{c}$$

$$\tan v = \frac{a}{b}$$

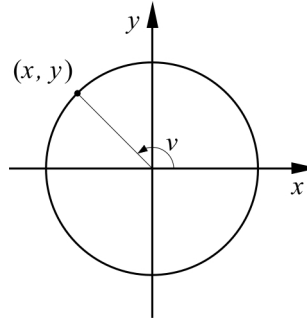


Unit circle

$$\sin v = y$$

$$\cos v = x$$

$$\tan v = \frac{y}{x}$$



Sine rule

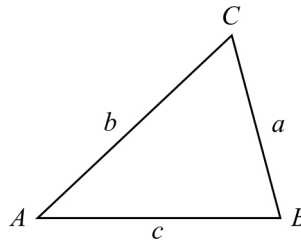
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area

$$T = \frac{ab \sin C}{2}$$



Trigonometric formulas

$$\sin^2 v + \cos^2 v = 1$$

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\sin 2v = 2 \sin v \cos v$$

$$\cos 2v = \begin{cases} \cos^2 v - \sin^2 v & (1) \\ 2 \cos^2 v - 1 & (2) \\ 1 - 2 \sin^2 v & (3) \end{cases}$$

$$a \sin x + b \cos x = c \sin(x + v) \text{ where } c = \sqrt{a^2 + b^2} \text{ and } \tan v = \frac{b}{a}$$

Circle equation

$$(x - a)^2 + (y - b)^2 = r^2$$

Exact values

| | | | | | | | | | |
|------------------------|-----------|----------------------|----------------------|----------------------|-----------------|----------------------|-----------------------|-----------------------|-------------|
| Angle v (degrees) | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
| (radians) | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| $\sin v$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\cos v$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{\sqrt{3}}{2}$ | -1 |
| $\tan v$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | Not def. | $-\sqrt{3}$ | -1 | $-\frac{1}{\sqrt{3}}$ | 0 |