

Formulas for mathematics 5

Algebra

Rules

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Quadratic equations

$$x^2 + px + q = 0$$

$$ax^2 + bx + c = 0$$

$$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Binomial theorem

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n} b^n$$

Arithmetic

Prefixes

T	G	M	k	h	d	c	m	μ	n	p
tera	giga	mega	kilo	hecto	deci	centi	milli	micro	nano	pico
10^{12}	10^9	10^6	10^3	10^2	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}

Powers

$$a^x a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^{-x} = \frac{1}{a^x}$$

$$a^x b^x = (ab)^x$$

$$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^0 = 1$$

Logarithms

$$y = 10^x \Leftrightarrow x = \lg y$$

$$y = e^x \Leftrightarrow x = \ln y$$

$$\lg x + \lg y = \lg xy$$

$$\lg x - \lg y = \lg \frac{x}{y}$$

$$\lg x^p = p \cdot \lg x$$

Absolute value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Functions and relations

Linear function

$$y = kx + m \quad k = \frac{y_2 - y_1}{x_2 - x_1}$$

$ax + by + c = 0$, where a and b are not both zero

Quadratic function

$$y = ax^2 + bx + c \quad a \neq 0$$

Power function

$$y = C \cdot x^a$$

Exponential function

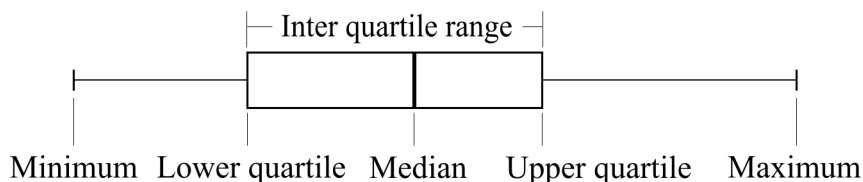
$$y = C \cdot a^x \quad a > 0 \text{ och } a \neq 1$$

Statistics and probability

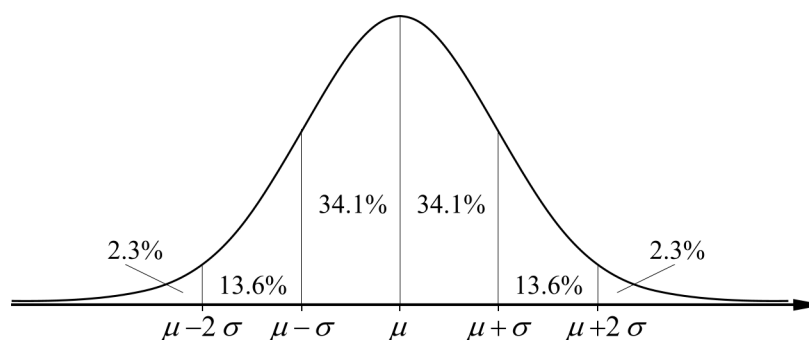
Standard deviation of a sample

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}}$$

Box plot



Normal distribution



Density function of the normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Differential and integral calculus

Definition of the derivative

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Derivative

Function	Derivative
x^n where n is a real number	nx^{n-1}
a^x ($a > 0$)	$a^x \ln a$
$\ln x$ ($x > 0$)	$\frac{1}{x}$
e^x	e^x
e^{kx}	$k \cdot e^{kx}$
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$1 + \tan^2 x = \frac{1}{\cos^2 x}$
$k \cdot f(x)$	$k \cdot f'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)}$ ($g(x) \neq 0$)	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

Chain rule

If $y = f(z)$ and $z = g(x)$ are two differentiable functions then it holds for $y = f(g(x))$ that

$$y' = f'(g(x)) \cdot g'(x) \text{ or } \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

Antiderivatives

Function	Antiderivatives
k	$kx + C$
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln x + C \quad (x > 0)$
e^x	$e^x + C$
e^{kx}	$\frac{e^{kx}}{k} + C$
$a^x \quad (a > 0, a \neq 1)$	$\frac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$

Complex numbers

Representation $z = x + iy = re^{iv} = r(\cos v + i \sin v)$ where $i^2 = -1$

Argument $\arg z = v \quad \tan v = \frac{y}{x}$

Absolute value $|z| = r = \sqrt{x^2 + y^2}$

Conjugate If $z = x + iy$ then $\bar{z} = x - iy$

Rules $z_1 z_2 = r_1 r_2 (\cos(v_1 + v_2) + i \sin(v_1 + v_2))$

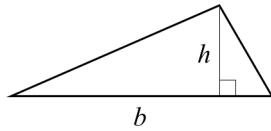
$$\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos(v_1 - v_2) + i \sin(v_1 - v_2))$$

de Moivre's formula $z^n = (r(\cos v + i \sin v))^n = r^n (\cos nv + i \sin nv)$

Geometry

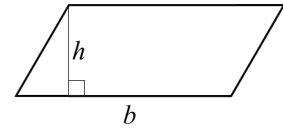
Triangle

$$A = \frac{bh}{2}$$



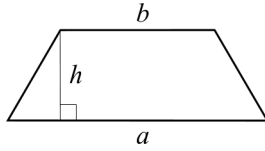
Parallelogram

$$A = bh$$



Trapezium

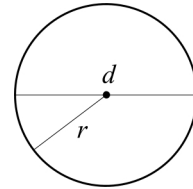
$$A = \frac{h(a+b)}{2}$$



Circle

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

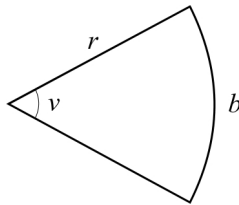
$$O = 2\pi r = \pi d$$



Circle sector

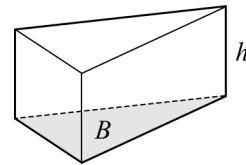
$$b = \frac{v}{360^\circ} \cdot 2\pi r$$

$$A = \frac{v}{360^\circ} \cdot \pi r^2 = \frac{br}{2}$$



Prism

$$V = Bh$$

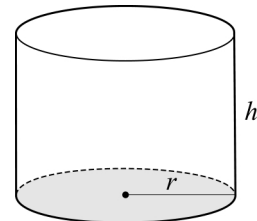


Cylinder

$$V = \pi r^2 h$$

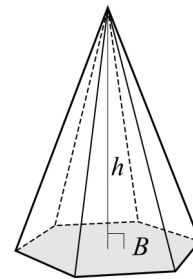
Lateral surface area

$$A = 2\pi r h$$



Pyramid

$$V = \frac{Bh}{3}$$

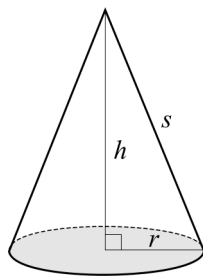


Cone

$$V = \frac{\pi r^2 h}{3}$$

Lateral surface area

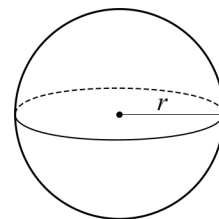
$$A = \pi r s$$



Sphere

$$V = \frac{4\pi r^3}{3}$$

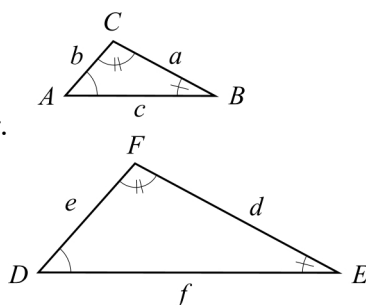
$$A = 4\pi r^2$$



Similarity

The triangles ABC and DEF are similar.

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$



Scale

Area scale factor = (Length scale factor)²

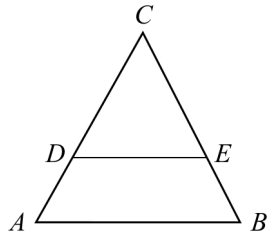
Volume scale factor = (Length scale factor)³

Triangle with a transversal line

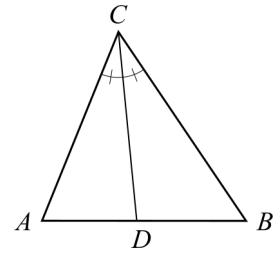
If DE is parallel to AB then

$$\frac{DE}{AB} = \frac{CD}{AC} = \frac{CE}{BC} \text{ and}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$

**Angle bisector theorem**

$$\frac{AD}{BD} = \frac{AC}{BC}$$

**Angles**

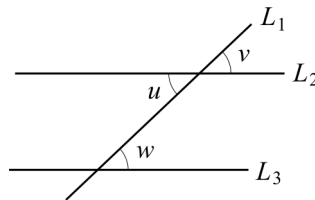
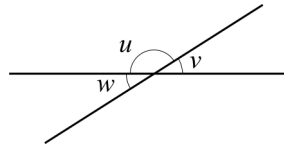
$u + v = 180^\circ$ Supplementary angles

$w = v$ Vertical angles

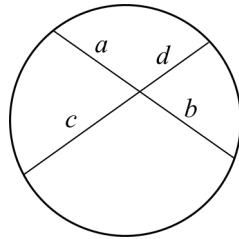
L_1 cuts two parallel lines L_2 and L_3

$v = w$ Corresponding angles

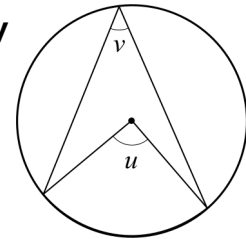
$u = w$ Alternate angles

**Chord theorem**

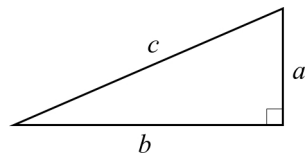
$$ab = cd$$

**Angles subtended by the same arc**

$$u = 2v$$

**Pythagoras' theorem**

$$a^2 + b^2 = c^2$$

**Distance formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint formula

$$x_m = \frac{x_1 + x_2}{2} \text{ and } y_m = \frac{y_1 + y_2}{2}$$

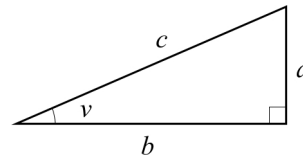
Trigonometry

Definitions

$$\sin v = \frac{a}{c}$$

$$\cos v = \frac{b}{c}$$

$$\tan v = \frac{a}{b}$$

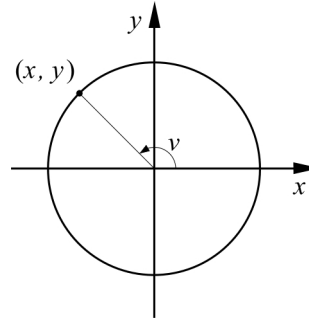


Unit circle

$$\sin v = y$$

$$\cos v = x$$

$$\tan v = \frac{y}{x}$$



Sine rule

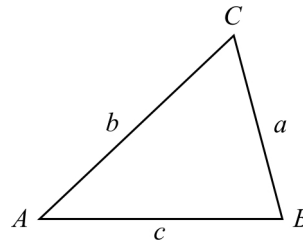
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area

$$T = \frac{ab \sin C}{2}$$



Trigonometric formulas

$$\sin^2 v + \cos^2 v = 1$$

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\sin 2v = 2 \sin v \cos v$$

$$\cos 2v = \begin{cases} \cos^2 v - \sin^2 v & (1) \\ 2 \cos^2 v - 1 & (2) \\ 1 - 2 \sin^2 v & (3) \end{cases}$$

$$a \sin x + b \cos x = c \sin(x + v) \text{ where } c = \sqrt{a^2 + b^2} \text{ and } \tan v = \frac{b}{a}$$

Circle equation

$$(x - a)^2 + (y - b)^2 = r^2$$

Exact values

Angle v (degrees)	0°	30°	45°	60°	90°	120°	135°	150°	180°
(radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Set theory

$$A \cap B = \{x | x \in A \text{ and } x \in B\} \quad A \cup B = \{x | x \in A \text{ or } x \in B\}$$

$$A \setminus B = \{x | x \in A \text{ and } x \notin B\} \quad A^c = \{x | x \in G \text{ and } x \notin A\}$$

Number theory**Congruence**

$a \equiv b \pmod{n}$ if the difference $a - b$ is divisible by n

If $a_1 \equiv b_1 \pmod{c}$ and $a_2 \equiv b_2 \pmod{c}$ then

1. $a_1 + a_2 \equiv b_1 + b_2 \pmod{c}$
2. $a_1 \cdot a_2 \equiv b_1 \cdot b_2 \pmod{c}$

If $a \equiv b \pmod{c}$ then

3. $m \cdot a \equiv m \cdot b \pmod{c}$ for all integers m
4. $a^n \equiv b^n \pmod{c}$ for all integers $n \geq 0$

Arithmetic series $s_n = n \cdot \frac{a_1 + a_n}{2}$, where $a_n = a_1 + (n-1) \cdot d$

Geometric series $s_n = a_1 \frac{k^n - 1}{k - 1}$, where $a_n = a_1 \cdot k^{n-1}$

Combinatorics

Permutations $P(n, k) = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-k+1) = \frac{n!}{(n-k)!}$ where $0 \leq k \leq n$

Combinations $C(n, k) = \binom{n}{k} = \frac{P(n, k)}{k!} = \frac{n!}{k!(n-k)!}$ where $0 \leq k \leq n$