

<b>Part B</b>	Problems 1-11 which only require answers.
<b>Part C</b>	Problems 12-16 which require complete solutions.
<b>Test time</b>	120 minutes for part B and C together.
<b>Resources</b>	Formula sheet and ruler.

### Level requirements

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 63 points of which 24 E-, 21 C- and 18 A-points.

Level requirements for test grades

E: 17 points

D: 25 points of which 7 points on at least C-level

C: 32 points of which 12 points on at least C-level

B: 42 points of which 6 points on A-level

A: 50 points of which 11 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled “*Only answers required*” you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thoughts and, where necessary, draw figures.

**Write your name, date of birth and educational program on all the sheets you hand in.**

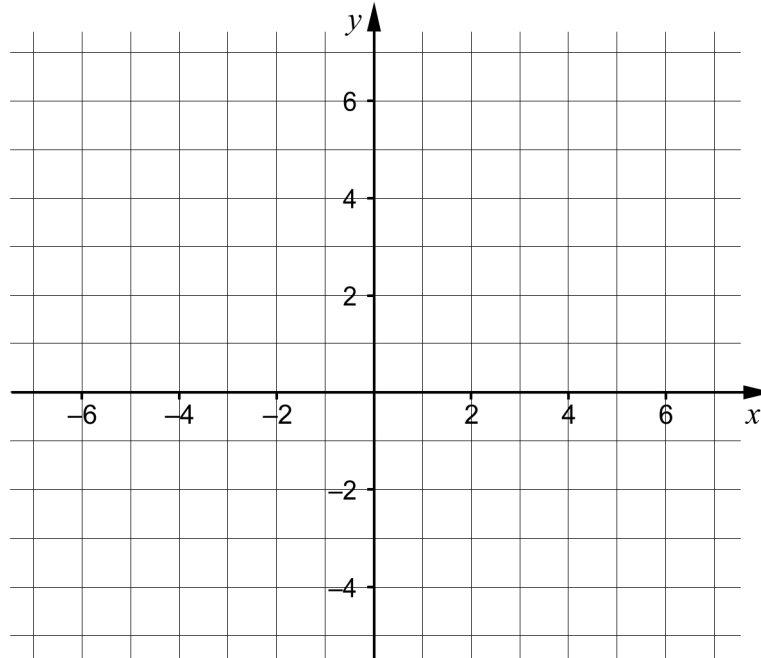
Name: \_\_\_\_\_

Date of birth: \_\_\_\_\_

Educational program: \_\_\_\_\_

**Part B:** Digital resources are not allowed. Only answer is required. Write your answers in the test booklet.

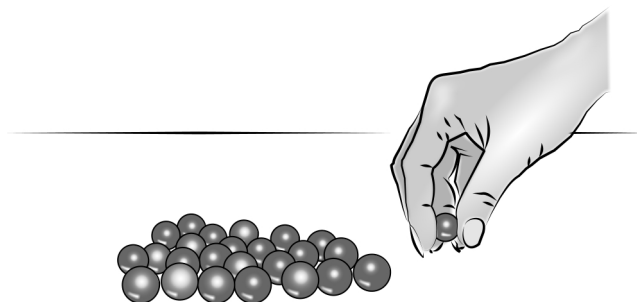
1.



- a) Draw the line  $y = 2x + 1$  in the coordinate system. (1/0/0)
- b) Give an example of an equation of another line that is parallel to the line in task a). \_\_\_\_\_ (1/0/0)

2. Hanna is going to order beads from the internet site Fina-PärLAN. A packet of beads costs SEK 15. A flat fee for postage and packing is added to every order.

- a) Hanna orders 5 packets of beads and pays SEK 125. How much was the fee for postage and packing? \_\_\_\_\_ (1/0/0)
- b) Find an expression for the total cost if Hanna orders  $x$  packets of beads. \_\_\_\_\_ (1/0/0)



3. Simplify  $(x + 3)^2 - x^2$  as far as possible. \_\_\_\_\_ (1/0/0)

4. Calculate  $25^{1/2}$  \_\_\_\_\_ (1/0/0)

5. Solve the equation  $x^2 - 4x = 0$  \_\_\_\_\_ (1/0/0)

6. What expression should be within the brackets in order for the equality to be true?

$x^2 - 16 = (x - 4) \cdot ( \quad )$  \_\_\_\_\_ (0/1/0)

7. Three figures consisting of dots are shown below. The figures are formed according to a pattern. More figures can be formed according to the same pattern.

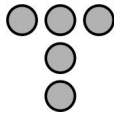


Figure 1

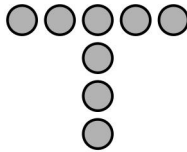


Figure 2

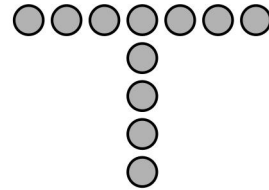


Figure 3

a) How many dots would there be in Figure 4? \_\_\_\_\_ (1/0/0)

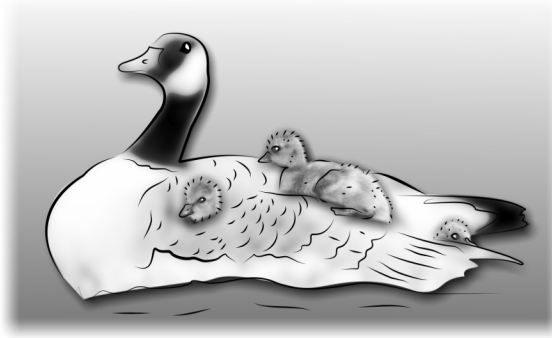
b) Find an expression for the number of dots in Figure  $n$ .  
\_\_\_\_\_ (0/1/0)

8. What should be written in the box in order for the linear system of equations

$$\begin{cases} 2x + 5y = 35 \\ \square x + 3y = 21 \end{cases} \text{ to have an infinite number of solutions?}$$

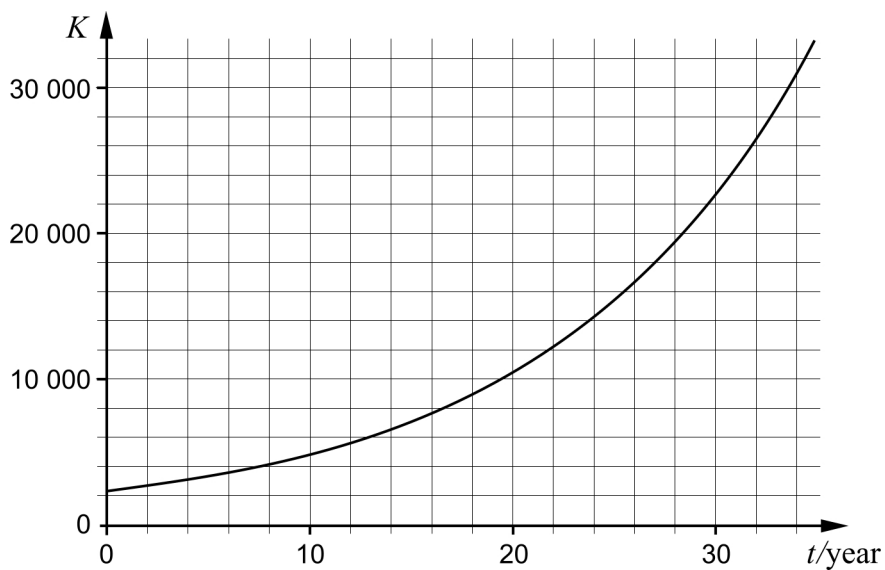
\_\_\_\_\_ (0/0/1)

9.



The Canada Goose was introduced into Sweden in the 1930s. The population has increased ever since. Every year, at the same time, there is a survey of the number of Canada Geese. The growth of the population can be described by an exponential model.

The diagram below shows the number of Canada Geese  $K$  as a function of time  $t$  years, where  $t = 0$  corresponds to the year 1977.



a) Use the graph and determine an approximate value of  $K(22)$

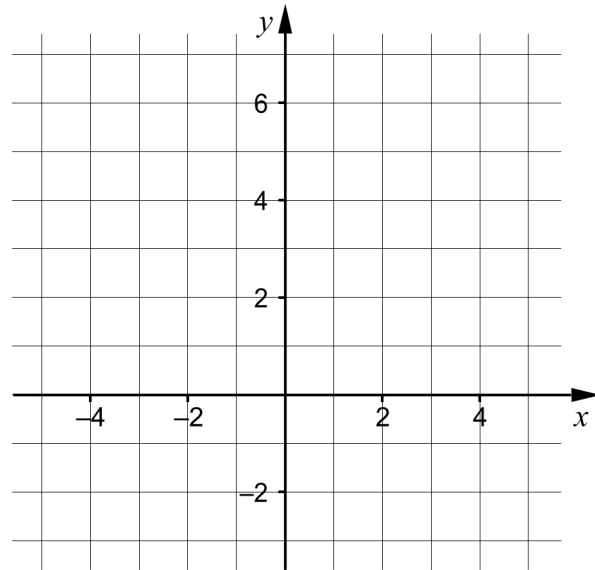
\_\_\_\_\_ (1/0/0)

b) Use the graph to determine in which year the number of Canada Geese reached 26 000

\_\_\_\_\_ (0/1/0)

10. It holds for the function  $f$  that:

- $f(-2) = 3$
- $f(x) = 0$  for  $x = 4$
- The domain of the function is  $-3 \leq x \leq 4$
- The range of the function is  $0 \leq f(x) \leq 5$



Draw a possible graph of the function  $f$  in the coordinate system.

(0/2/1)

11. Simplify the expression  $3^{\frac{n}{2}-1} + 3^{\frac{n}{2}-1} + 3^{\frac{n}{2}-1}$  as far as possible.

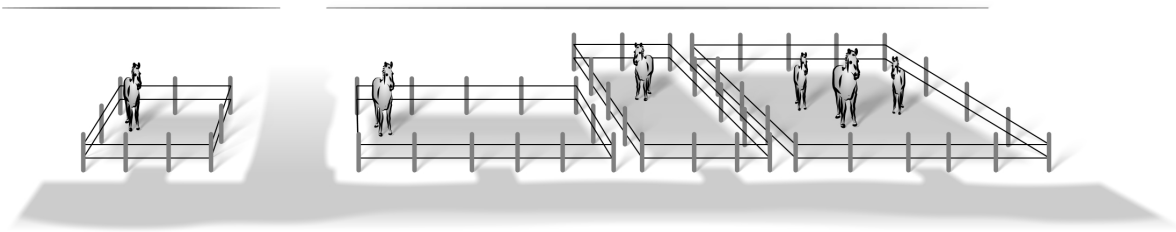
\_\_\_\_\_ (0/0/1)

**Part C:** Digital resources are not allowed. Do your solutions on separate sheets of paper.

12. Solve the equation  $x^2 + 2x - 24 = 0$  algebraically. (2/0/0)

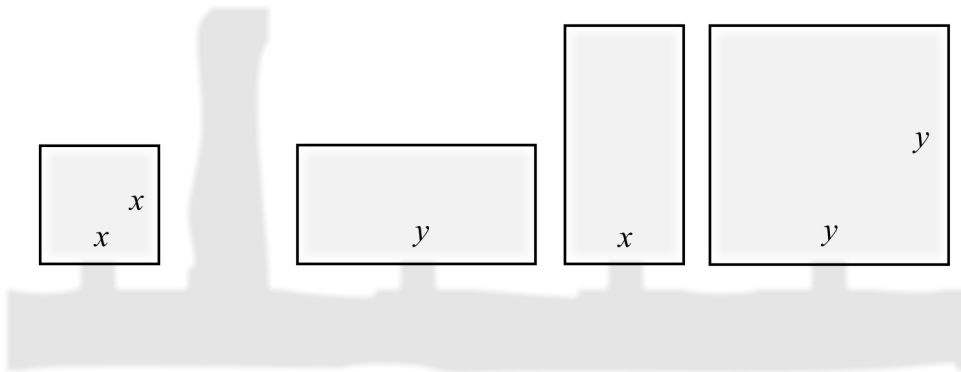
13. Solve the simultaneous equations  $\begin{cases} 4y + x = 20 \\ y - 2x = -13 \end{cases}$  algebraically. (2/0/0)

14. The figure show four pastures that are quadratic and rectangular respectively with side lengths  $x$  and  $y$  metres.



Below is a sketch of the pastures seen from above.

(m)



The horses will be moved into a new common pasture. The new pasture is quadratic and the area is equal to the total area of all the four original pastures combined.

Find a simplified expression for the length of the side of the new pasture. (0/1/1)

15. Elin and Sanna are discussing a mathematical problem with two propositions,  $P$  and  $Q$ , where

$$P: x > 2$$

$$Q: x^2 > 4$$

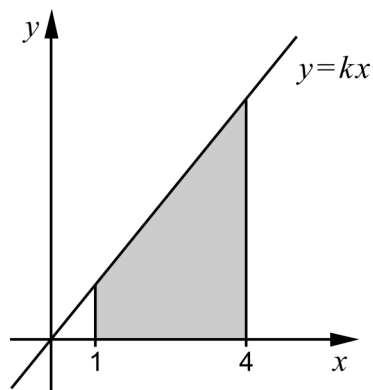
Elin claims that: "For these propositions, it holds that  $P \Rightarrow Q$ "

Sanna replies: "No, I think it is the other way round,  $Q \Rightarrow P$ "

Who is right? Justify your answer.

(0/1/0)

16. A region is bounded by the  $x$ -axis, the lines  $x = 1$  and  $x = 4$  and the straight line  $y = kx$  where  $k > 0$



Calculate the gradient  $k$  algebraically so that the area of the region is exactly 10 area units.

(0/0/4)

<b>Part D</b>	Problems 17-24 which require complete solutions.
<b>Test time</b>	120 minutes.
<b>Resources</b>	Digital resources, formula sheet and ruler.

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**Write your name, date of birth and educational program on all the sheets you hand in.**

Name: \_\_\_\_\_

Date of birth: \_\_\_\_\_

Educational program: \_\_\_\_\_



**Part D:** Digital resources are allowed. Do your solutions on separate sheets of paper.

17. Find the equation of the straight line that passes through the points (1, 7) and (5, 15) (2/0/0)

18. At Floda weather station the outside temperature is measured every hour. The measurements during one night in October, can according to a simplified model, be described by the quadratic function

$$f(x) = 0.5x^2 - 3.75x + 6$$

where  $f(x)$  represents the temperature expressed in °C and  $x$  represents the number of hours after midnight (at 00:00).

- a) Calculate  $f(2)$  (1/0/0)
- b) Interpret what  $f(4) = -1$  means in this context. (0/1/0)
- c) At what time does the lowest temperature occur according to the model? (0/2/0)
19. The table shows the price list for two different mobile phone subscriptions:

Subscription All-prat		Subscription Prata-på	
Subscription fee	SEK 299 / month	Subscription fee	SEK 199 / month
Data rates (down)	10 Mbit/s	Surfing speed	Up to 3 Mbit/s
Data rates (up)	4.6 Mbit/s	Surfing volume 1	Free within Sweden
Free surf/month	10 GB/month		
Talk rate	SEK 0.29 / minute	Talk rate	SEK 0.69 / minute

Victor wants to compare the two subscriptions and investigate the monthly cost.

- a) Write the monthly cost as a function of the call time  $x$  minutes for subscription All-prat and Prata-på respectively. (2/0/0)
- b) Help Victor investigate which subscription is the cheapest one depending on the length of his call time during one month. (1/2/0)

20. At the start of 2004, Niklas bought a flat for SEK 635 000. He sold it 7 years later for SEK 1 115 000.
- a) Assume that the increase in value was exponential during that period of time. Calculate the yearly percentage increase in value for the flat. (0/2/0)
- b) What would be the value of the flat at the start of 2020 if the increase in value continued at the same pace? (0/2/0)



21. Lisa says to Melker:

- Think of a number between  $-100$  and  $100$ .
- Square the number.
- Subtract your original number 18 times.
- Add 50.

Lisa: Which number did you get?

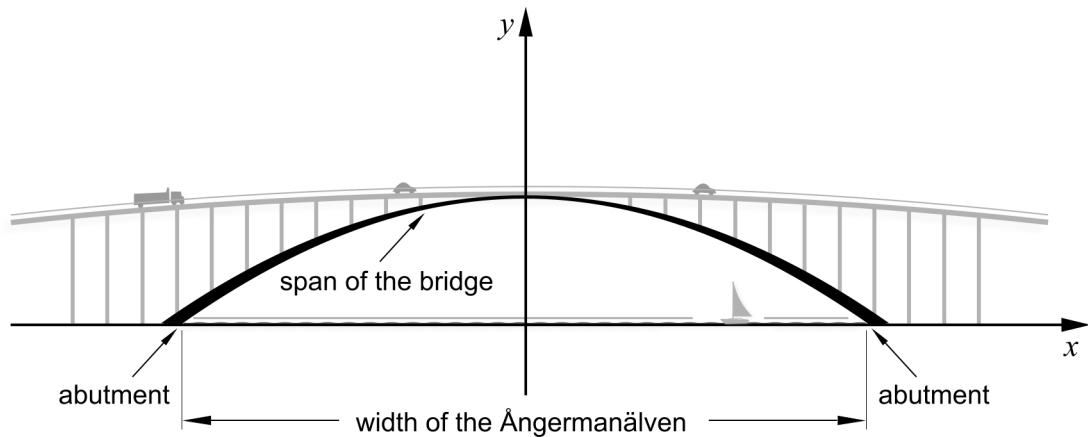
Melker: I got 5

Lisa: Did you think of 15?

Melker: No.

What number did Melker think of? (Assuming that he has calculated correctly.) (0/0/2)

22. The Sandö bridge is a bridge crossing the Ångermanälven river. The bridge was built in 1943 and was until 1964 the world's longest single-span concrete arch bridge.



The shape of the arch can be described by the quadratic function  $y = h(x)$  where

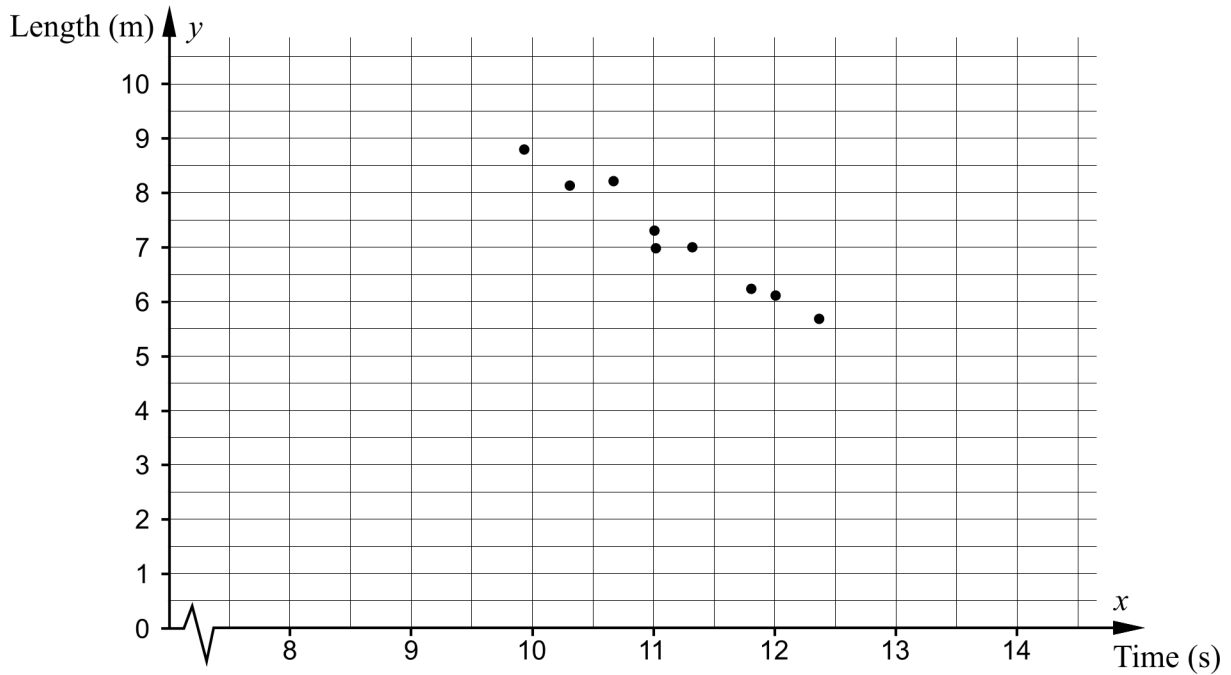
$$h(x) = -0.0023x^2 + 40$$

$h(x)$  is the height above the water in metres.

$x$  is the distance in metres from the middle of the bridge along the surface of the water.

- a) How high above the water are the cars when they pass the highest point of the bridge? *Only answer required* (1/0/0)
- b) A 15-metre-high sailing boat is going to pass under the bridge. How close to one of the bridge abutments can the boat pass? (0/0/3)

23. Nine people competing in both the long jump and the 100 metres present their best results. These results are marked in the diagram below. The diagram shows that there seems to be a linear relationship between the length of a jump and the time on the 100 metres.

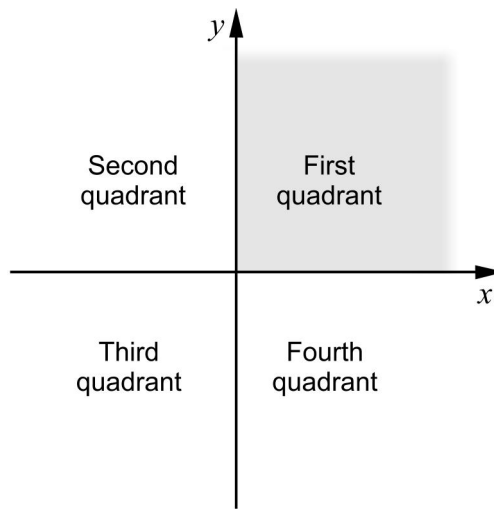


- a) Draw a straight line that, as good as possible, shows the relation between the length of a jump and the time on the 100 metres. Determine the equation of this line on the form  $y = kx + m$  (0/2/0)

The relationship can be seen as a model of how the length of a jump depends on the time of a 100-metre race.

- b) Usain Bolt holds the world record on the 100 metres with a time of 9.58 seconds. How far would Usain Bolt be able to jump in the long jump according to this model? (1/0/0)
- c) Give your comments on whether there is any limitation to the model. (0/1/0)

24. The two straight lines  $y = ax - 2$  and  $y = x - 1$ , where  $a$  is a constant, intersect in the first quadrant.



Investigate the possible values of the constant  $a$ .

(0/1/2)

### **To the student - Information about the oral part**

You will be given a problem that you will solve in writing, and then you will present your solution orally. If you need, you can ask your classmates and your teacher for help when solving the problem. Your oral presentation starts with you presenting what the problem is about and then you describe and explain your solution. You must present all steps in your solution. However, if you have done the same calculation several times (for example in a table) it might be sufficient if you present some of the calculations. Your presentation should take a maximum of 5 minutes, and be held to a smaller group of your classmates and one or more teachers.

The problem given to you should, on the whole, be solved algebraically. You might need a calculator to do some of the calculations but, when presenting your solution, you should avoid referring to the use of your calculator for drawing graphs and/or symbolic handling (if that is the type of calculator you are using).

When assessing your oral presentation, the teacher will take into consideration:

- how complete, relevant and structured your presentation is,
- how well you describe and explain the train of thought behind your solution,
- how well you use the mathematical terminology.

#### *How complete, relevant and structured your presentation is*

Your presentation must contain the necessary parts in order for a listener to follow and understand your thoughts. What you say should be in a suitable order and be relevant. The listener must understand how calculations, descriptions, explanations and conclusions are connected with each other.

#### *How well you describe and explain the train of thought behind your solution*

Your presentation should contain both descriptions and explanations. To put it simple, a description answers the question *how* and an explanation answers the question *why*. You describe something when you for instance tell *how* you have done a calculation. You explain something when you for instance justify *why* you could use a certain formula.

#### *How well you use the mathematical terminology*

In your presentation you should use a language that contains mathematical terms, expressions and symbols, suitable for the problem you have solved.

Mathematical terms are for example words like “exponent”, “function” and “graph”.

An example of a mathematical expression is that  $x^2$  is read “ $x$  to the power 2” or “ $x$  squared”. Some examples of mathematical symbols are  $\pi$  and  $f(x)$ , which are read “pi” and “ $f$  of  $x$ ”.

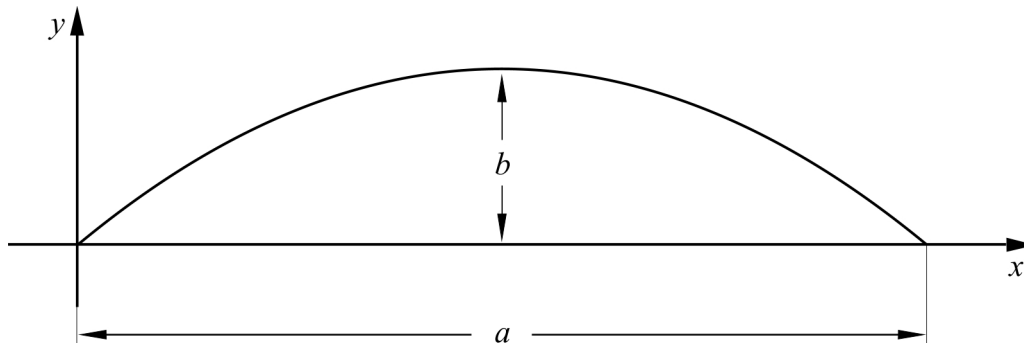
**Problem 1. Quadratic function**

Name: \_\_\_\_\_

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The figure below shows the graph to the quadratic function  $y = 3x - x^2$



- a) What is the length of distance  $a$ ?
- b) What is the length of distance  $b$ , that is the distance between the highest point of the curve and the  $x$ -axis?



**Problem 2. School equipment**

Name: \_\_\_\_\_

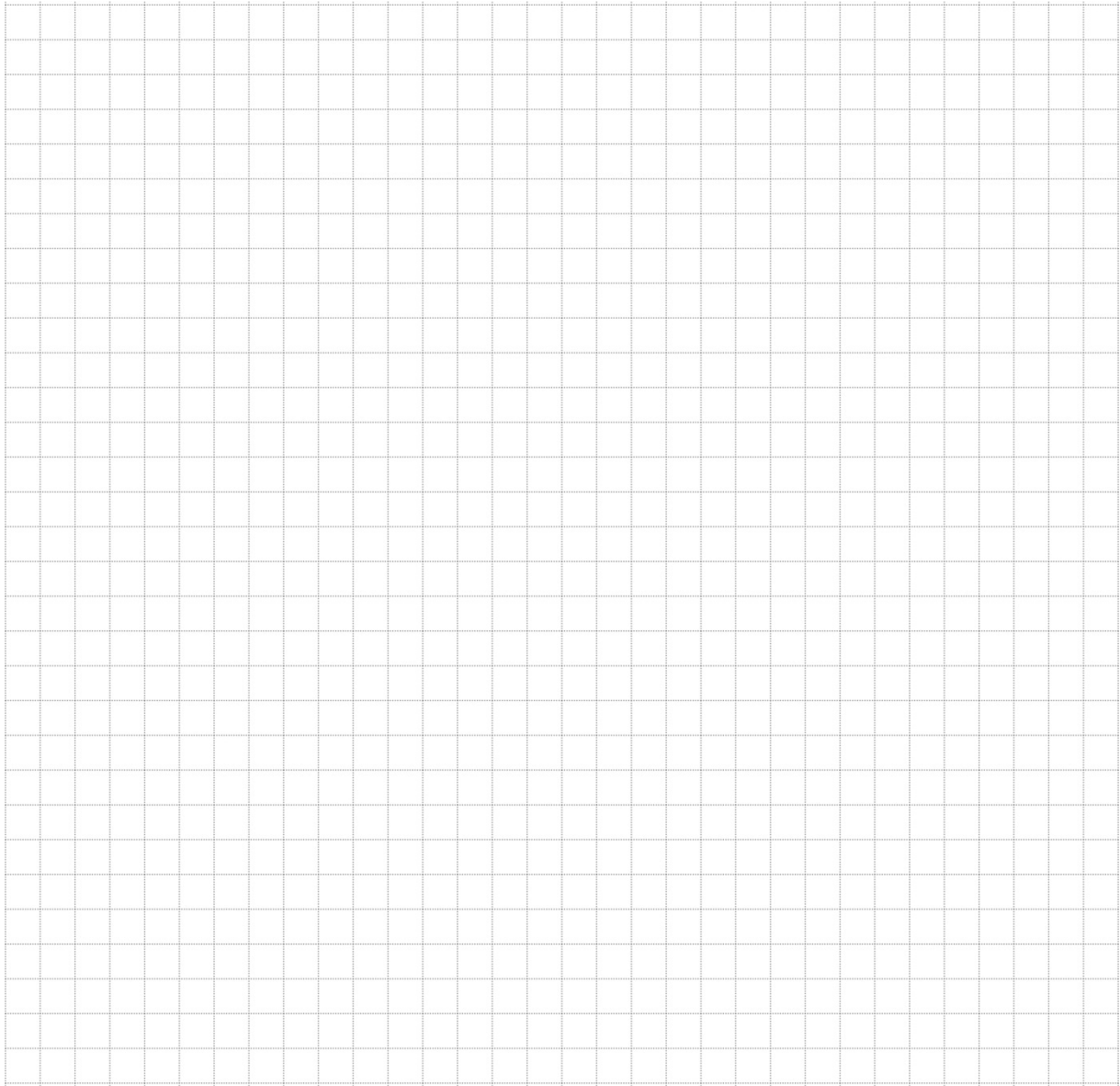
**When assessing your oral presentation, the teacher will take into consideration:**

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School is about to start so Hanna and Lukas go to the book shop to buy note books and other school equipment. The book shop sells note books for SEK 12 each but also pencils and rubbers. Hanna buys four note books, three pencils and six rubbers and pays SEK 78.

Lukas buys seven note books, eight pencils and two rubbers and pays SEK 122.

What is the price of a pencil and a rubber respectively?





**Problem 3. Magic square**

Name: \_\_\_\_\_

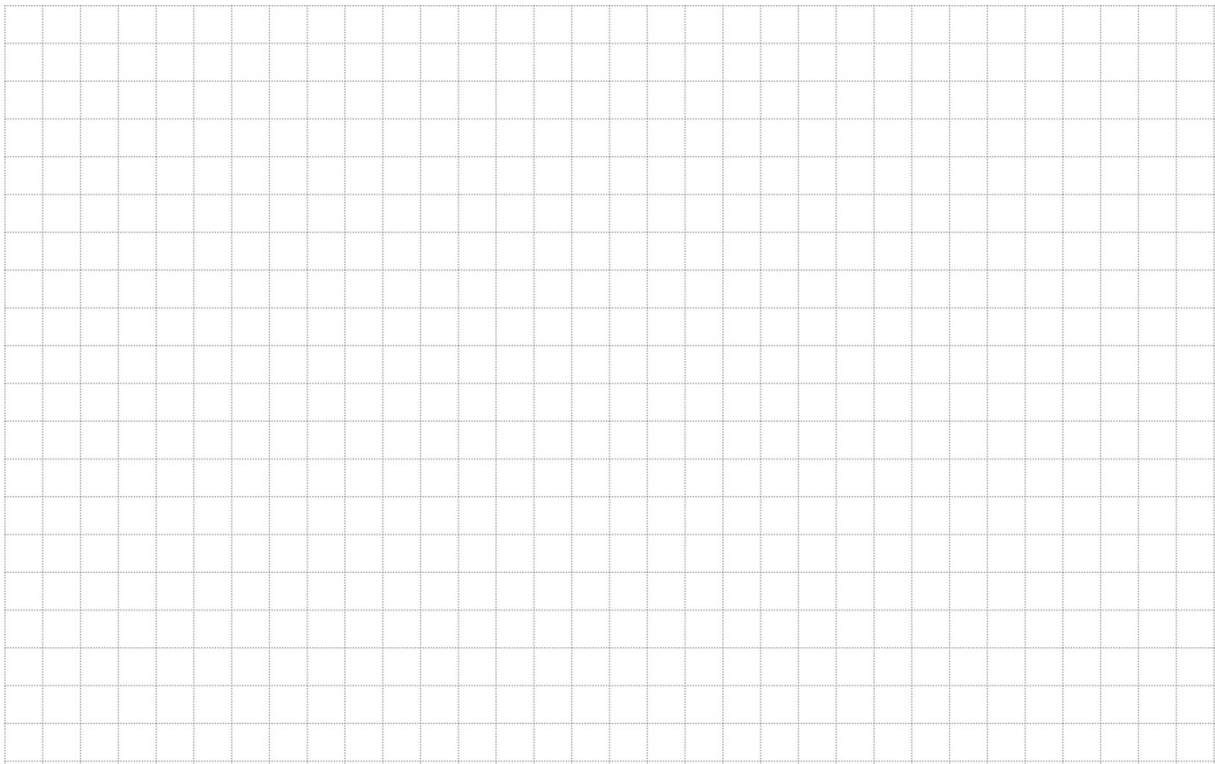
**When assessing your oral presentation, the teacher will take into consideration:**

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- how well you use the mathematical terminology.

In a magic square, the sum of the numbers in the squares is equal for every row, every column and every diagonal. In the square below different expressions have been printed in some of the squares.

$x^2-20$	?	?
$x^2$	$3x+2$	$x-2$
$x+2$	?	?

- Find the positive  $x$ -value that makes the values of the expressions in the filled-in squares satisfy the requirements of a magic square.
- Calculate the values for each one of the nine squares and then draw the complete magic square.



**Problem 4. Women’s maximum pulse rate**

Name: \_\_\_\_\_

**When assessing your oral presentation, the teacher will take into consideration:**

- how complete, relevant and structured your presentation is,
- how well you describe and explain the train of thought behind your solution,
- how well you use the mathematical terminology.

A group of women are part of a study that investigates how the women’s maximum pulse rates vary with their age. The women are 15 years old the first time their maximum pulse rate is measured. Two further measurements are made when the women are 30 years old and 40 years old respectively.

Age $x$ (years)	Maximum pulse rate $y$ (beats/minute)
15	194
30	182
40	174

The table shows values from Lisa, one of the women in the group.

- a) Investigate if the values in the table form a linear relationship.
- b) Determine, with the aid of the table, an algebraic relationship of how Lisa’s maximum pulse rate  $y$  beats/minute depends on age  $x$  years and use your relationship to determine at what age she has a maximum pulse rate of 146 beats/minute.

**Bedömningsmatris för bedömning av muntlig kommunikativ förmåga**

<b>Kommunikativ förmåga</b>	<b>E</b>	<b>C</b>	<b>A</b>	<b>Max</b>
<p><b><i>Fullständighet, relevans och struktur</i></b></p> <p>Hur fullständig, relevant och strukturerad elevens redovisning är.</p>	<p>Redovisningen kan sakna något steg eller innehålla något ovidkommande.</p> <p>Det finns en övergripande struktur men redovisningen kan bitvis vara fragmentarisk eller rörig.</p> <p>(1/0/0)</p>		<p>Redovisningen är fullständig och endast relevanta delar ingår.</p> <p>Redovisningen är välstrukturerad.</p> <p>(1/0/1)</p>	(1/0/1)
<p><b><i>Beskrivningar och förklaringar</i></b></p> <p>Förekomst av och utförlighet i beskrivningar och förklaringar.</p>	<p>Någon förklaring förekommer men tyngdpunkten i redovisningen ligger på beskrivningar.</p> <p>Utförligheten i de beskrivningar och de förklaringar som framförs kan vara begränsad.</p> <p>(1/0/0)</p>		<p>Redovisningen innehåller tillräckligt med utförliga beskrivningar och förklaringar.</p> <p>(1/0/1)</p>	(1/0/1)
<p><b><i>Matematisk terminologi</i></b></p> <p>Hur väl eleven använder matematiska termer, symboler och konventioner.</p>	<p>Eleven använder matematisk terminologi med rätt betydelse vid enstaka tillfällen i redovisningen.</p> <p>(1/0/0)</p>	<p>Eleven använder matematisk terminologi med rätt betydelse och vid lämpliga tillfällen genom delar av redovisningen.</p> <p>(1/1/0)</p>	<p>Eleven använder matematisk terminologi med rätt betydelse och vid lämpliga tillfällen genom hela redovisningen.</p> <p>(1/1/1)</p>	(1/1/1)
<b>Summa</b>				(3/1/3)