Part B	Problems 1-10 which only require answers.
Part C	Problems 11-15 which require complete solutions.
Test time	120 minutes for Part B and Part C together.
Resources	Formula sheet and ruler.

Level requirements

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 64 points consisting of 23 E-, 22 C- and 19 A-points.

Level requirements for test grades
E: 16 points
D: 25 points of which 7 points on at least C-level
C: 33 points of which 13 points on at least C-level
B: 43 points of which 6 points on A-level
A: 52 points of which 11 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled "*Only answers required*" you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: ____

Date of birth: ____

Educational programme: ________________________________

Part B: Digital resources are not allowed. *Only answer is required*. Write your answers in the test booklet.

1. It holds for the polynomial function f that $f(x) = 3x^4 + 7x^2 + 3$

a) What is the degree of the function f? ____________________ (1/0/0)

b) Determine $f'(x)$. ____________________ (1/0/0)

2. Specify two different antiderivatives of $f(x) = 7x + 4$

____________________ and ____________________ (2/0/0)

3. During the first seconds after starting the distance a car travels can be described with $s(t) = 3t + t^2$ where *s* is the distance in metres and *t* is the time in seconds.

Determine the speed *v* of the car as a function of time *t*.

$v(t) =$ ____________________ (1/0/0)

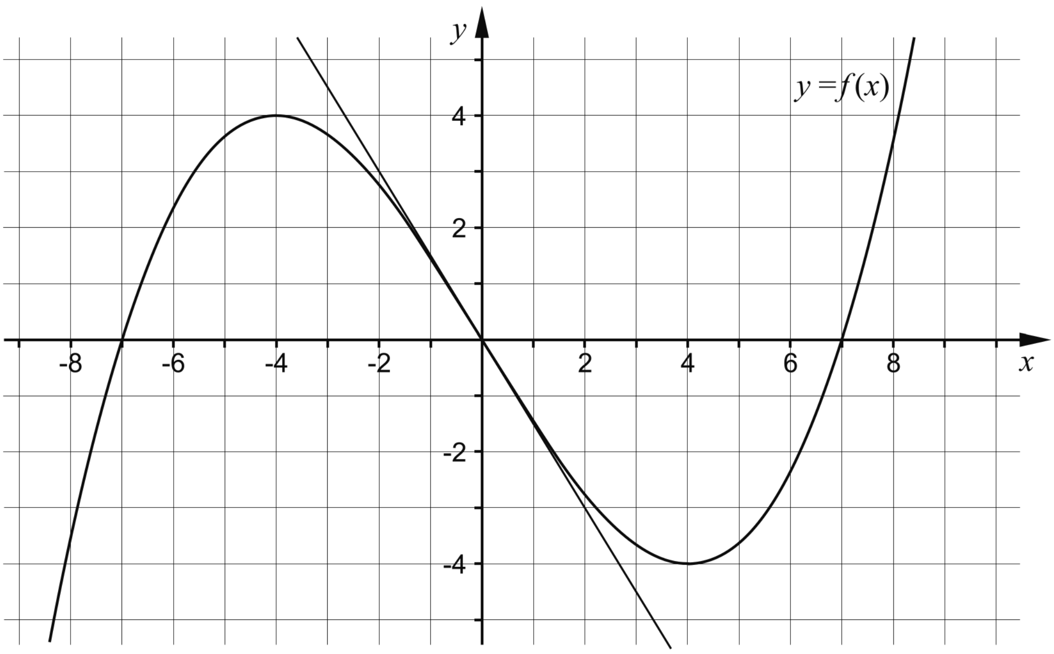
4. Solve the equation $(x+2)(x-3)(x+4) = 0$ ____________________ (1/0/0)

5. Simplify the following expressions as far as possible.

a) $16 + (x^3 + 4)(x^3 - 4)$ ____________________ (1/0/0)

b) $\dfrac{x}{(x+4)^9} + \dfrac{4}{(x+4)^9}$ ____________________ (0/1/0)

6. The figure below shows the graph of a cubic function f and a tangent which touches the graph at the origin.



a) Determine the zeroes of the derivative. ________________________ (1/0/0)

b) Determine $f'(0)$. ________________________ (1/0/0)

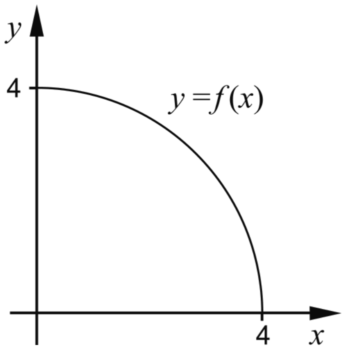
c) Sketch the graph of the derivative of the function in the coordinate system above. (0/1/1)

7. The geometric sum $2-2\cdot1.5+2\cdot1.5^2-2\cdot1.5^3+...-2\cdot1.5^{19}$ equals one of the alternatives A-H below. Which?

A. $2\cdot\frac{(-1.5)^{18}-1}{-1.5-1}$	B. $2\cdot\frac{(-1.5)^{19}-1}{-1.5-1}$	C. $2\cdot\frac{(-1.5)^{20}-1}{-1.5-1}$	D. $2\cdot\frac{(-1.5)^{21}-1}{-1.5-1}$
E. $2\cdot\frac{1.5^{18}-1}{1.5-1}$	F. $2\cdot\frac{1.5^{19}-1}{1.5-1}$	G. $2\cdot\frac{1.5^{20}-1}{1.5-1}$	H. $2\cdot\frac{1.5^{21}-1}{1.5-1}$

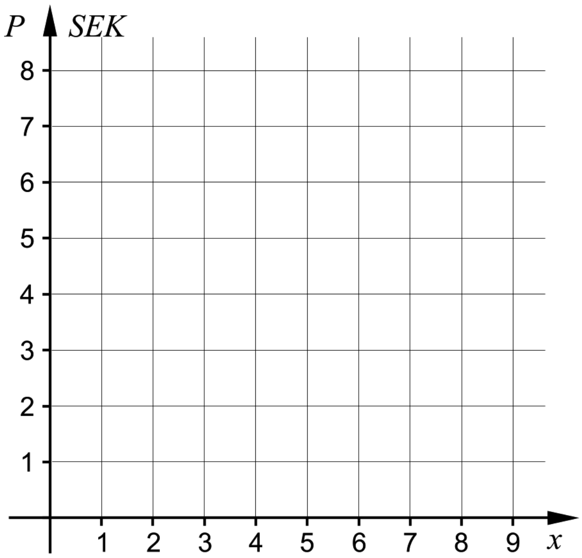
________________________ (0/1/0)

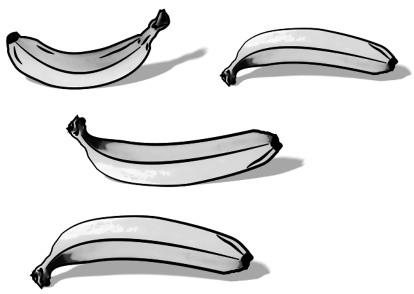
8. The graph of the function f forms a quarter circle in the first quadrant.



Evaluate $\int_0^4 f(x)\,dx$. Give an exact answer. ______________________ (0/1/0)

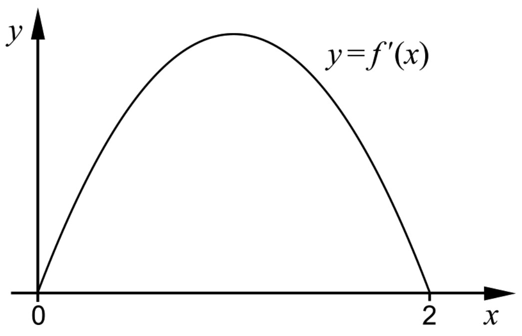
9. The price of bananas at the cafeteria of Hagaskolan is SEK 2 a piece. The price SEK P is a function of the number of bananas x. Draw the graph of the function within the interval $1 \leq x \leq 4$ in the coordinate system below.





(0/1/0)

10. The function f has the derivative f'. The figure below shows the graph of f'. Which of the statements A-F is *always* true?



A. $f(2)$ is positive

B. $f(2) - f(0)$ is positive

C. $f(1)$ is zero

D. $f(0)$ is zero

E. $f(1) - f(2)$ is positive

F. $f(0) - f(1)$ is positive ______________________ (0/0/1)

Part C: Digital resources are not allowed. Do your solutions on separate sheets of paper.

11. The figures A and B below show the graphs of two cubic functions.

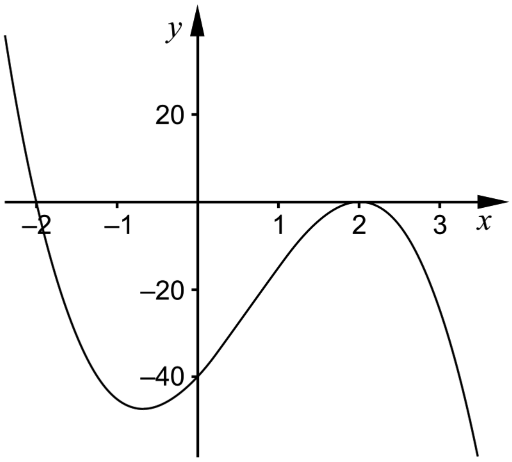


Figure A

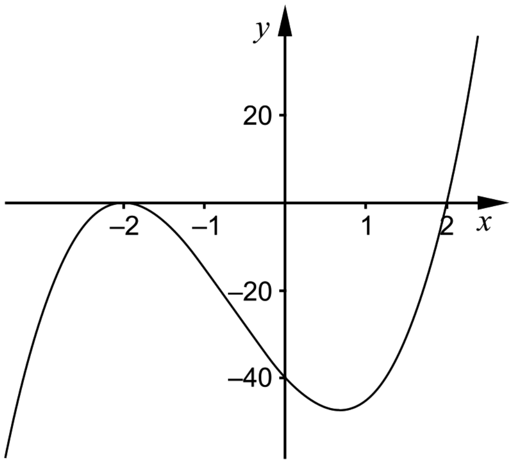


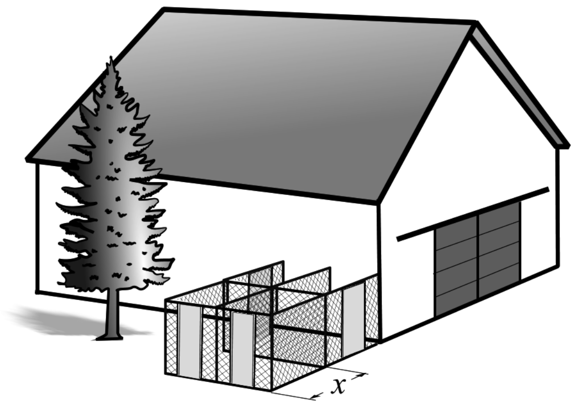
Figure B

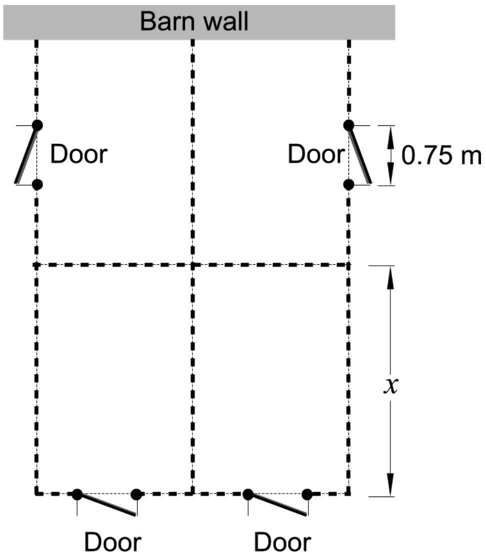
a) Which of the figures shows the graph of a cubic function f where $f'(2)=0$? Justify your answer. (1/0/0)

b) Which of the figures shows the graph of $f(x)=5(x-2)(x+2)^2$? Justify your answer. (0/1/0)

12. Karin is going to build four rectangular shaped dog runs for her dogs. All four dog runs will have the same dimensions and will be enclosed by fencing.

Karin has 45 m of fencing and four doors that she will use for the dog runs. Two of the dog runs will be built against a barn wall. Therefore, fencing will not be needed on the side made by the barn wall. The doors are 0.75 m wide, with the same height as the fencing and will be placed according to the figure.





The area of each of the dog runs is given by the function $A(x) = 12x - 1.5x^2$ where A is the area in m^2 and x is the length in m of one of the sides of the dog run, se figure.

a) Use the derivative to find the value of x that gives the largest possible area for each dog run. (2/0/0)

b) Show that the area of a dog run is given by the function $A(x) = 12x - 1.5x^2$ (0/0/3)

13. Solve the equation $\dfrac{6}{x-3} - \dfrac{18}{x(x-3)} = 2$ (0/3/0)

14. Evaluate $\displaystyle\int_0^4 e^{\frac{x}{2}}\,dx$. Give an exact answer. (0/2/0)

15. Determine the constant A so that $\displaystyle\lim_{x\to\infty} \frac{Ax}{4x + A} = \frac{1}{7}$ (0/0/3)

Part D	Problems 16-24 which require complete solutions.
Test time	120 minutes.
Resources	Digital resources, formula sheet and ruler.

Level requirements

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 64 consisting of 23 E-, 22 C- and 19 A-points.

Level requirements for test grades
E: 16 points
D: 25 points of which 7 points on at least C-level
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For problems labelled "*Only answers required*" you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures and show how you use your digital resources.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: ____________________

Date of birth: ____________________

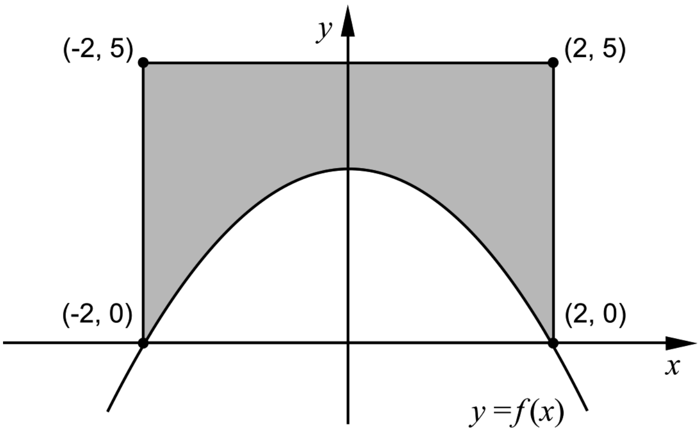
Educational programme: ____________________

Part D: Digital resources are allowed. Do your solutions on separate sheets of paper.

16. Determine the values of x for which it holds that the graph of $f(x) = x^3 - 0.88x$ has the gradient 5 (2/0/0)

17. Solve the equation $x(x^2 - 5) = 5(2 - x)$ (2/0/0)

18. The figure below shows the graph of $f(x) = -0.75x^2 + 3$ and a rectangle. The rectangle has its corners at the points (–2, 0), (–2, 5), (2, 0) and (2, 5).



a) Use the figure and explain, in words, why $\int_{-2}^{0} f(x)\,dx = \int_{0}^{2} f(x)\,dx$ (1/1/0)

b) Calculate the area of the shaded region. (2/1/0)

19. Andrea is going to start saving money by depositing a certain amount of money into a bank account at the beginning of each year. She wants to save up SEK 50 000 for a trip. She plans to make her first deposit at the beginning of the year 2014 and the last one at the beginning of the year 2020. She counts on the yearly interest rate being 2% for the whole period of time.

What amount should she deposit every time if she wants there to be SEK 50 000 in her account immediately after the last deposit? (0/2/0)

20. Today there are approximately 7 billion people on Earth. A model that describes the number of people on Earth as a function of time is

$$N(t) = \frac{11}{1 + 3.4e^{-0.03 \cdot t}}$$

where *N* is the number of people in billions and *t* is the time in years after 1950.

a) Calculate the number of people on Earth in 1950. (1/0/0)

b) According to the model, the number of people on Earth will in time reach an upper limit. Use the model and determine this upper limit for the number of people. (0/3/0)

21. It holds for a function f that $f(x) = (x-a)(x-b)$ where *a* and *b* are constants. Find the relation that must be true for *a* and *b* in order for the graph of f to have a tangent with gradient 2 when $x = 4$ (0/3/0)

22. It holds for the polynomial function f that $f'(x) > 0$ for all *x*. Investigate how many real solutions there are to the equation $f(x) = 0$ (0/0/2)

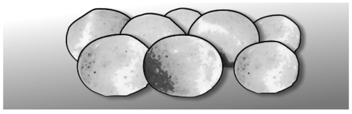
23. Albin's weight can be described by the function

$$V(t) = 0.10t^3 - 1.23t^2 + 6.51t + 3.72$$

where the weight *V* kg is a function of the time *t* years after his birth. The function is true for his first eight years in life.

The rate of increase of Albin's weight varies. Calculate what values the rate of increase can have during the first eight years of Albin's life. (0/0/2)

24. Anton and Amanda have been given the task of baking buns and biscuits which they then will sell in order to get money for a school trip. They write down the two recipes on a piece of paper and decide that the profit should be SEK 4 per bun and SEK 2 per biscuit.



Recipe for 100 Buns

2400 grams flour
500 grams butter
----

180 grams sugar
2.5 packets yeast
1.5 litres milk
1 teaspoon salt

Profit: SEK 4 per bun

Recipe for 100 Biscuits

600 grams flour
500 grams butter
----

170 grams sugar
4 teaspoons baking powder
6 teaspoons vanilla sugar

Profit: SEK 2 per biscuit

Anton and Amanda want to make as large a profit as possible. At the same time, they think about whether they should bake both buns and biscuits or if it will be enough to bake just one of them. They count on selling everything they bake.

In order to know how much they can bake, they find out how much butter and flour they have at home. Together they have 4800 grams of flour and 1750 grams of butter.

Calculate the maximum profit Anton and Amanda can make on their baking. You only need to take into consideration how much flour and butter they will use. (0/0/4)

To the student - Information about the oral part

You will be given a problem that you will solve in writing, and then you will present your solution orally. If you need, you can ask your classmates, your teacher and your textbook for help when solving the problem. Your oral presentation starts with you presenting what the problem is about and then you describe and explain your solution. You must present all steps in your solution. However, if you have done the same calculation several times (for example in a table) it might be sufficient if you present some of the calculations. Your presentation should take a maximum of 5 minutes, and be held to a smaller group of your classmates and one or more teachers.

The problem given to you should, on the whole, be solved algebraically. You might need a calculator to do some of the calculations but, when presenting your solution, you should avoid referring to the use of your calculator for drawing graphs and/or symbolic handling (if that is the type of calculator you are using).

When assessing your oral presentation, the teacher will take into consideration:

- how complete, relevant and structured your presentation is,
- how well you describe and explain the train of thought behind your solution,
- how well you use the mathematical terminology.

How complete, relevant and structured your presentation is
Your presentation must contain the necessary parts in order for a listener to follow and understand your thoughts. What you say should be in a suitable order and be relevant. The listener must understand how calculations, descriptions, explanations and conclusions are connected with each other.

How well you describe and explain the train of thought behind your solution
Your presentation should contain both descriptions and explanations. To put it simple, a description answers the question *how* and an explanation answers the question *why*. You describe something when you for instance tell *how* you have done a calculation. You explain something when you for instance justify *why* you could use a certain formula.

How well you use the mathematical terminology
In your presentation you should use a language that contains mathematical terms, expressions and symbols, suitable for the problem you have solved.

Mathematical terms are for example words like “exponent”, “function” and “graph”.
An example of a mathematical expression is that x^2 is read “x to the power 2” or “x squared”.
Some examples of mathematical symbols are π and $f(x)$, which are read “pi” and “f of x”.

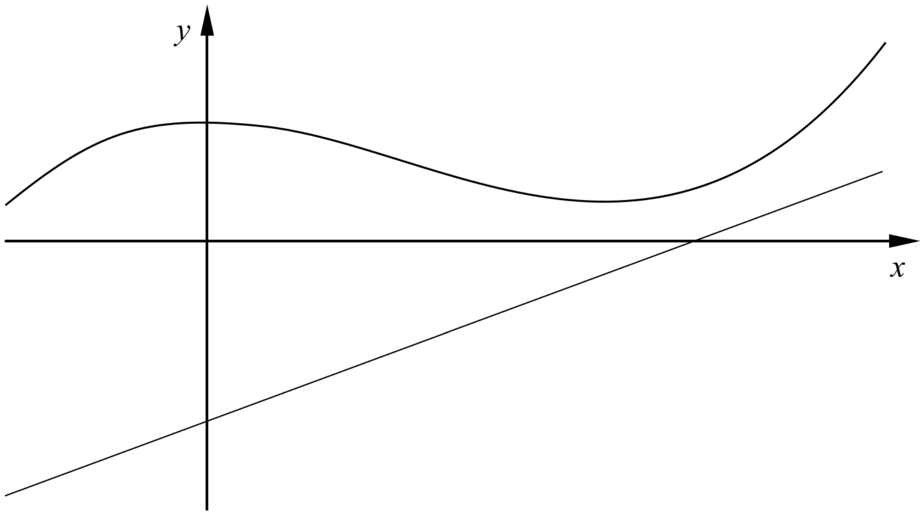
Problem 1.

Name:______________________________

When assessing your oral presentation, the teacher will take into consideration:

- how complete, relevant and structured your presentation is,
- how well you describe and explain the train of thought behind your solution,
- how well you use the mathematical terminology.

Find the shortest distance in the *y*-direction between the graphs of the functions f and g where $f(x) = 2x^3 - 6x^2 + 12$ and $g(x) = 7.5x - 18$ and $x > 0$

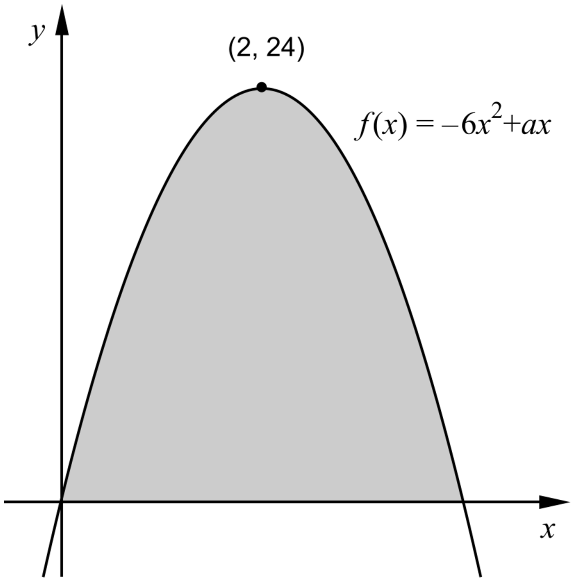


Problem 2.

Name:________________________________

When assessing your oral presentation, the teacher will take into consideration:
- how complete, relevant and structured your presentation is,
- how well you describe and explain the train of thought behind your solution,
- how well you use the mathematical terminology.

The function f is given as $f(x) = -6x^2 + ax$ where a is a constant. The graph of the function f has a maximum point (2, 24), see the figure.



Find the area of the shaded region which is enclosed by the graph of the function $f(x) = -6x^2 + ax$ and the x-axis.

Problem 3.

Name:________________________________

When assessing your oral presentation, the teacher will take into consideration:
- how complete, relevant and structured your presentation is,
- how well you describe and explain the train of thought behind your solution,
- how well you use the mathematical terminology.

In this problem you are going to investigate the function $v = 3x + y$
The two variables x and y satisfy the following conditions:

$$\begin{cases} 2x + y \geq 14 \\ x + 2y \leq 16 \\ y \geq 2 \end{cases}$$

Find the largest and the smallest value of the function $v = 3x + y$

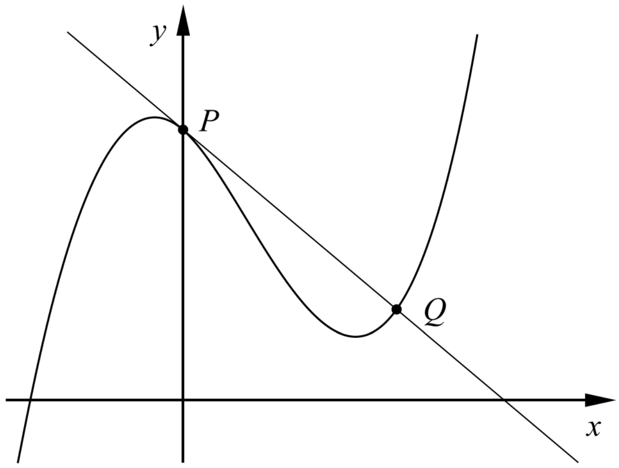
Problem 4.

Name:______________________________

When assessing your oral presentation, the teacher will take into consideration:

- how complete, relevant and structured your presentation is,
- how well you describe and explain the train of thought behind your solution,
- how well you use the mathematical terminology.

The figure shows the curve $y = 2x^3 - 4x^2 - 3x + 9$ and the tangent to the curve at the point *P* where $x = 0$. This tangent cuts the curve at another point *Q*.



Find the coordinates of the point *Q*.

Bedömningsmatris för bedömning av muntlig kommunikativ förmåga

Kommunikativ förmåga	E	C	A	Max
Fullständighet, relevans och struktur Hur fullständig, relevant och strukturerad elevens redovisning är.	Redovisningen kan sakna något steg eller innehålla något ovidkommande. Det finns en övergripande struktur men redovisningen kan bitvis vara fragmentarisk eller rörig. (1/0/0)		Redovisningen är fullständig och endast relevanta delar ingår. Redovisningen är välstrukturerad. (1/0/1)	(1/0/1)
Beskrivningar och förklaringar Förekomst av och utförlighet i beskrivningar och förklaringar.	Någon förklaring förekommer men tyngdpunkten i redovisningen ligger på beskrivningar. Utförligheten i de beskrivningar och de förklaringar som framförs kan vara begränsad. (1/0/0)		Redovisningen innehåller tillräckligt med utförliga beskrivningar och förklaringar. (1/0/1)	(1/0/1)
Matematisk terminologi Hur väl eleven använder matematiska termer, symboler och konventioner.	Eleven använder matematisk terminologi med rätt betydelse vid enstaka tillfällen i redovisningen. (1/0/0)	Eleven använder matematisk terminologi med rätt betydelse och vid lämpliga tillfällen genom delar av redovisningen. (1/1/0)	Eleven använder matematisk terminologi med rätt betydelse och vid lämpliga tillfällen genom hela redovisningen. (1/1/1)	(1/1/1)
Summa				(3/1/3)