Part B	Problems 1-10 which only require answers.
Part C	Problems 11-19 which require complete solutions.
Test time	150 minutes for Part B and Part C together.
Resources	Formula sheet and ruler.

Level requirements

The test consists of three written parts (Part B, Part C and Part D). Together they give a total of 62 points consisting of 22 E-, 23 C- and 17 A-points.

Level requirements for test grades
E: 15 points
D: 24 points of which 7 points on at least C-level
C: 32 points of which 13 points on at least C-level
B: 41 points of which 5 points on A-level
A: 49 points of which 9 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A- point.

For problems labelled "*Only answer required*" you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: ____

Date of birth: ____________________________________

Educational programme: ______________________________

Part B: Digital resources are not allowed. *Only answer is required*. Write your answers in the test booklet.

1. Specify a complex number z in the form $z = a + bi$ so that

a) $\operatorname{Im} z = 4$ ____________________ (1/0/0)

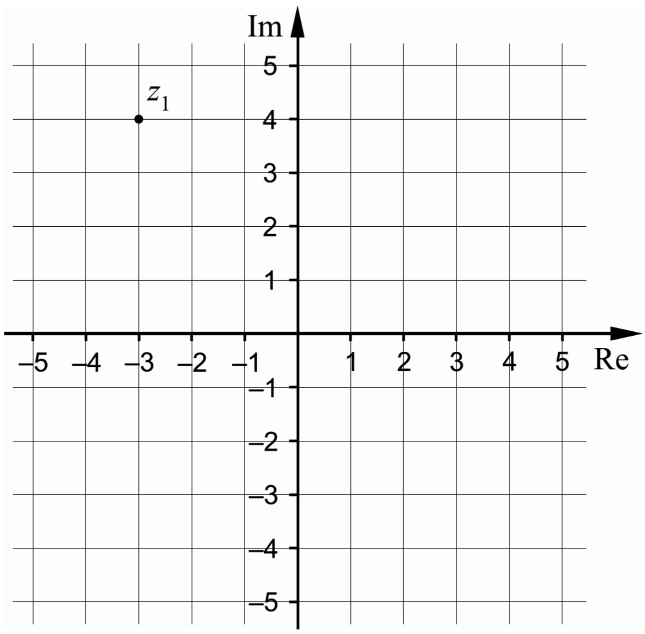
b) $\arg z = 45°$ ____________________ (1/0/0)

2. Differentiate

a) $f(x) = \cos 5x$ ____________________ (1/0/0)

b) $g(x) = x \cdot e^x$ ____________________ (1/0/0)

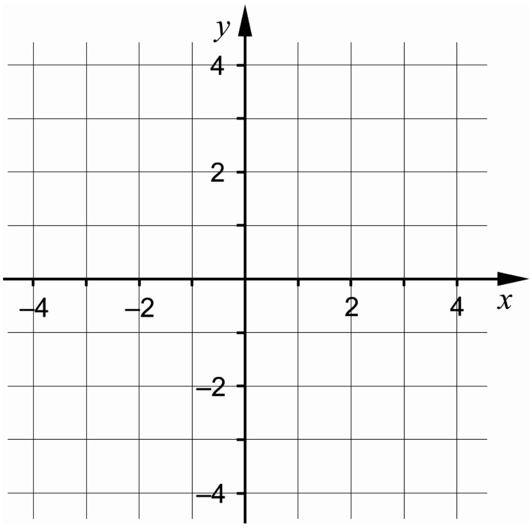
3. The figure below shows a complex plane where the number z_1 is marked.



a) Calculate $|z_1|$ ____________________ (1/0/0)

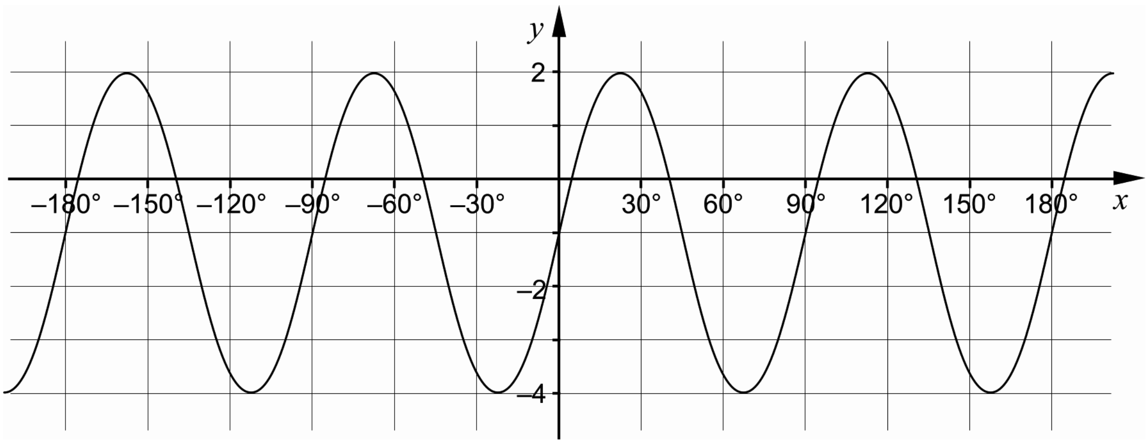
b) Mark the number $\bar{z}_2$ in the complex plane above when $z_2 = -5 - i$ (1/0/0)

4. a) Use the coordinate system below and mark a region whose area can be calculated by $\int_{-1}^{1}(3+x)\,dx$ (1/0/0)



b) Determine the value of $\int_{-1}^{1}(3+x)\,dx$ ____________________ (1/0/0)

5. The figure shows the graph of the function $y = A\sin kx + B$



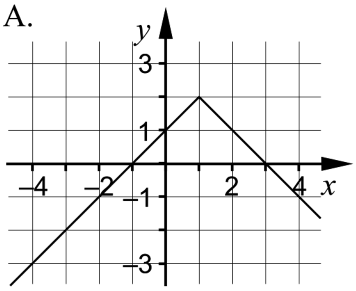
Determine the constants *A*, *B* and *k*

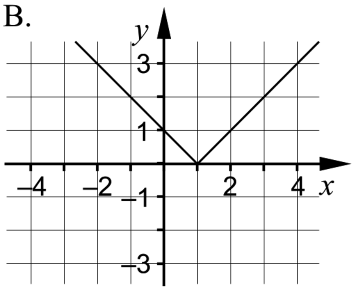
$A =$ ____________________

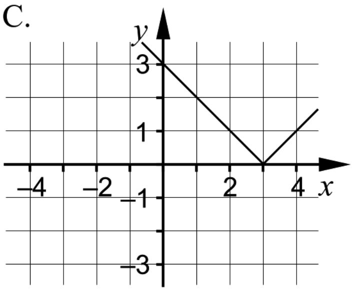
$B =$ ____________________

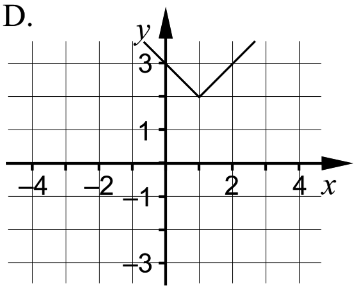
$k =$ ____________________ (1/1/0)

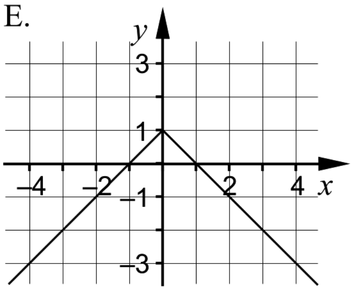
6. Which of the following figures A-H shows the graph of the function $f(x) = 2 - |x - 1|$?

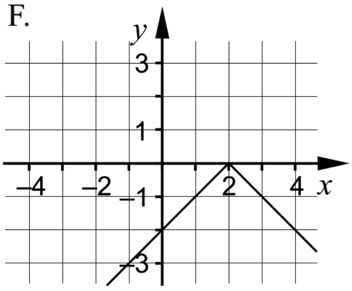
A.


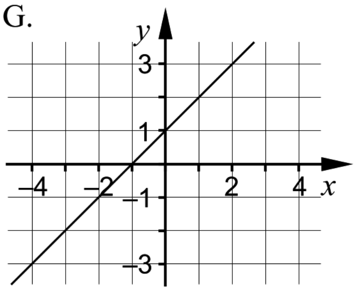
B.


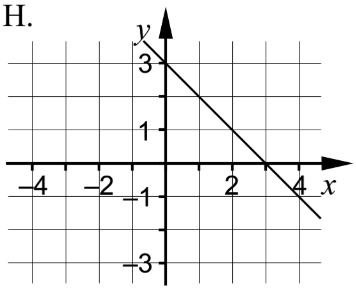
C.


D.


E.


F.


G.


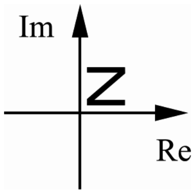
H.


________________________ (0/1/0)

7. Calculate $\lim_{h \to 0} \frac{f(h) - f(0)}{h}$ when $f(x) = 2x + \sin x$

________________________ (0/0/1)

8. A set of complex numbers that together form the letter Z is marked in the complex plane.

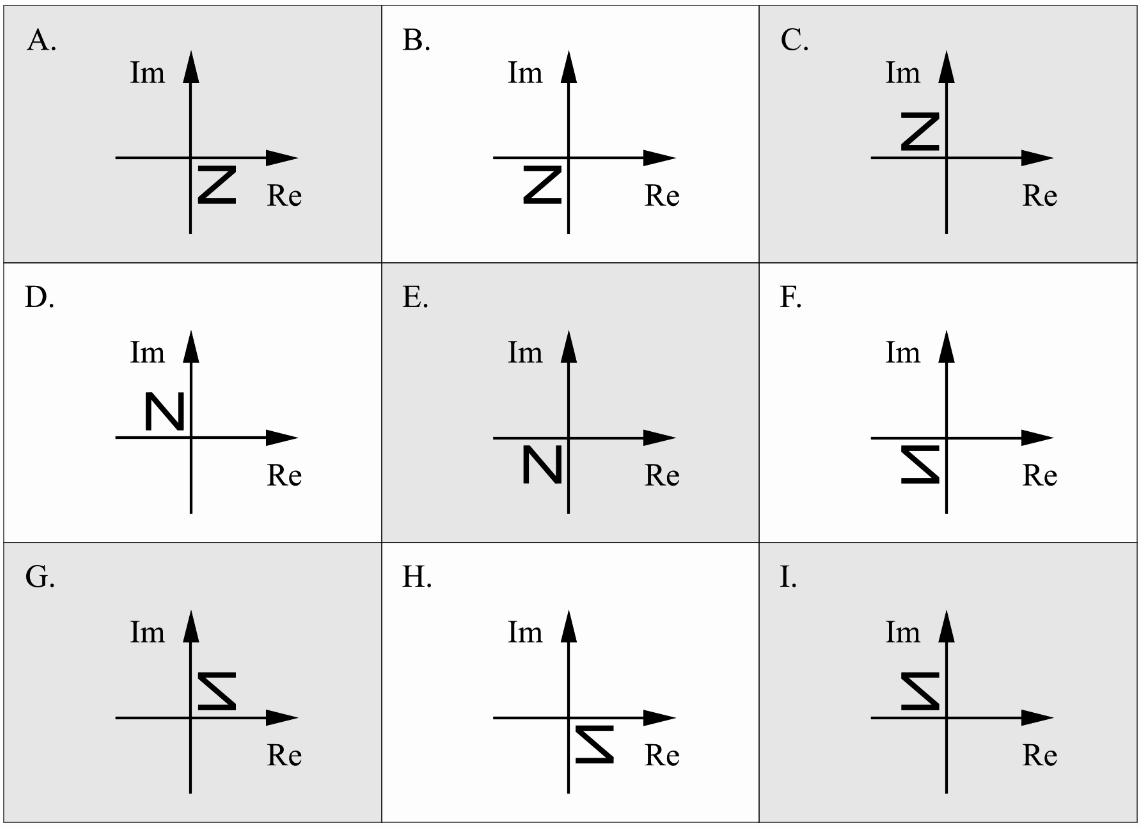


a) Which of the alternatives A-I below shows the figure formed by the conjugates to the numbers that form Z in the figure above?

______________________ (0/1/0)

b) Which of the alternatives A-I below shows the figure formed when the numbers forming Z in the original figure above are multiplied by i?

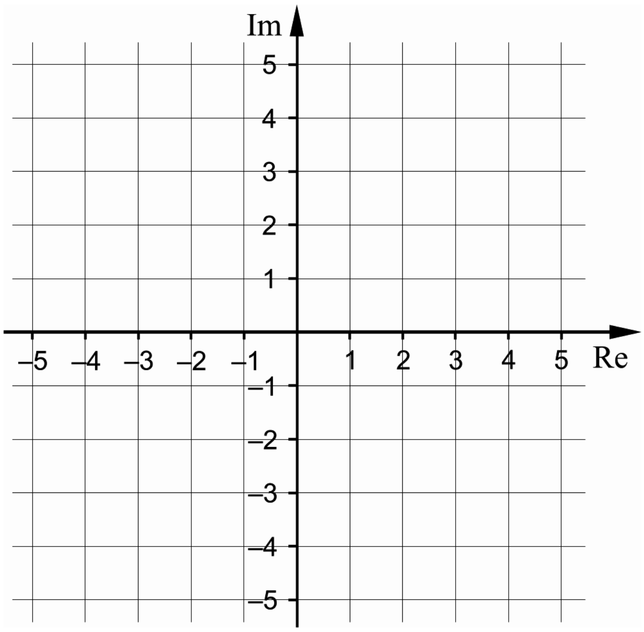
______________________ (0/0/1)



9. Write down a function f that has the derivative $f'(x) = x^2 \cdot e^{x^3+5}$

_______________________ (0/0/1)

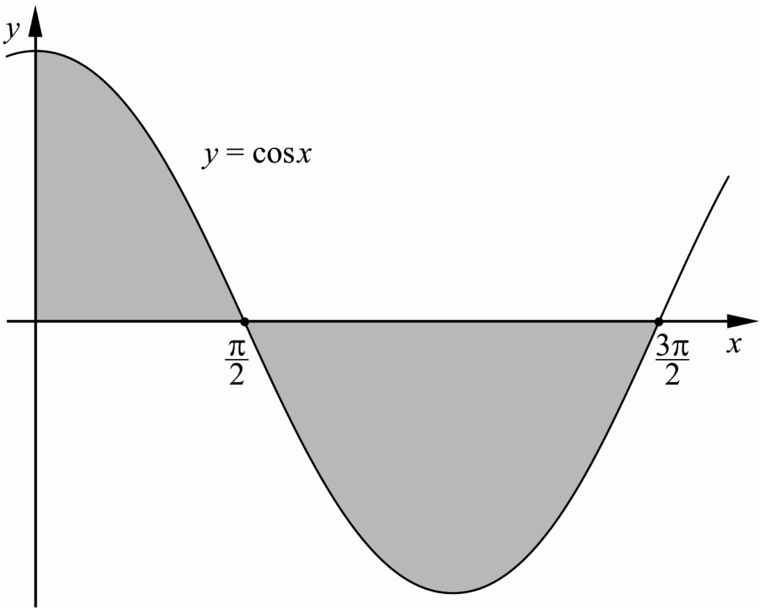
10. In the complex plane, mark the complex numbers z for which it holds that $|z-4| = |z-2i|$



(0/0/2)

Part C: Digital resources are not allowed. Write your solutions on separate sheets of paper.

11. Calculate the total area of the shaded regions in the figure below.



(2/0/0)

12. Show that $\sin^2 30° + \cos^2 30° = \sin^2 51° + \cos^2 51°$ (2/0/0)

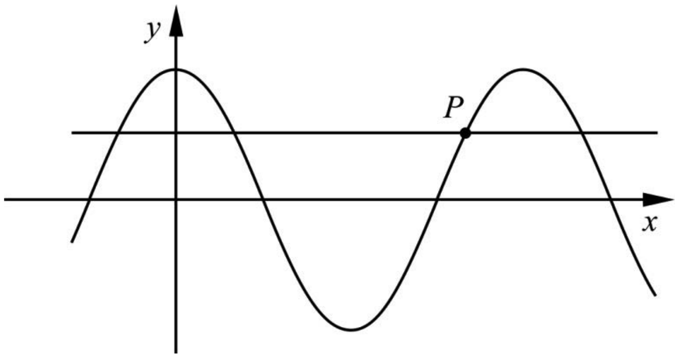
13. Determine the complex number $z = a + b\mathrm{i}$ so that $\bar{z} + 3z = \mathrm{i}z + 9$ (1/1/0)

14. Given the equation $x^3 + 2x^2 + x - 18 = 0$

a) Show that $x = 2$ is a root of the equation. (1/0/0)

b) Find the rest of the roots of the equation. (0/2/0)

15. The figure below shows the curve $y = \cos 2x$ and the line $y = \dfrac{1}{2}$



Find the x-coordinate of the intersection point P (1/2/0)

16. Solve the equation $z^3 + 27\mathrm{i} = 0$ (0/3/0)

17. It holds for the complex numbers z_1 and z_2 that $z_2 = z_1 \cdot (1 - \mathrm{i})$ and that z_1 lies within the area $45° < \arg z_1 < 135°$ in the complex plane.

In which area of the complex plane can you find the number z_2? (0/1/1)

18. Evaluate $\int_0^1 f''(x)\,\mathrm{d}x$ when $f(x) = \sin(\pi x^2)$ (0/1/2)

19. Show that the function $f(x) = x^3 + 3x^2 + ax$ has no maximum- or minimum points if $a \geq 3$ (0/1/3)

Part D	Problems 20-27 which require complete solutions.
Test time	120 minutes.
Resources	Digital resources, formula sheet and ruler.

Level requirements

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E: 15 points
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Date of birth: ______________________________

Educational programme: ______________________________

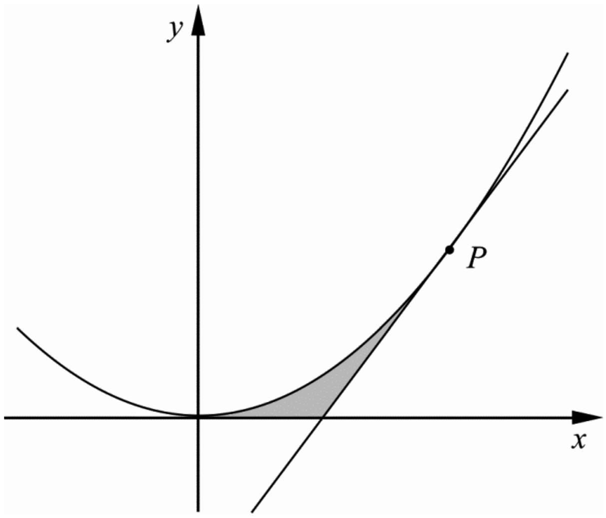
Part D: Digital resources are allowed. Write your solutions on separate sheets of paper.

20. The temperature in Haparanda was measured over 24 hours in July. According to a simplified model, the temperature during this time can be described by the relation $y = 15 + 5\sin(0.26x)$
where y °C is the temperature and x is the number of hours after 08.00.

Calculate the rate of change for the temperature at the time 12.00. (2/0/0)

21. Determine the number a so that $y = a \cdot e^{2x}$ is a solution to the differential equation $y' + y = e^{2x}$ (2/0/0)

22. The figure below shows the parabola $f(x) = x^2$ and the line $g(x) = 4x - 4$
The line touches the parabola at the point P. The parabola, the line and the x-axis together enclose the shaded region in the figure.



Calculate the area of the shaded region. (2/1/0)

23. Write down a function that has two vertical asymptotes.

Only answer is required (0/2/0)

24. As a part of a quality control of a bakery a number of cinnamon buns are weighed. The quality control shows that the weight is normally distributed with an average weight of 120 grams and a standard deviation of 4.0 grams.

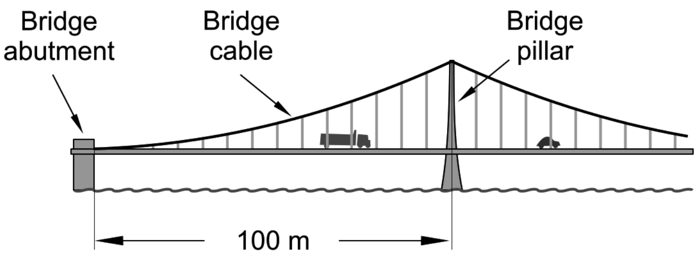
How many cinnamon buns can be expected to have a weight between 115 grams and 130 grams out of 450 cinnamon buns baked in one day? (0/2/0)

25. The shape of the bridge cable in the figure below can according to a simplified model be described by the function

$f(x) = 0.040x^{3/2}$ within the interval $0 \le x \le 100$, where

f is the height above the roadway in metres and

x is the distance in metres from the bridge abutment, measured along the roadway.



Facts:

The length s of a curve $y = f(x)$ within the interval $a \le x \le b$ is given by the relation

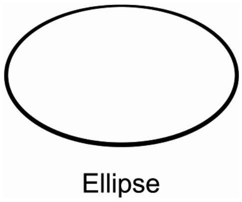
$$s = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$$

Calculate the length of the bridge cable between the bridge abutment and the bridge pillar. (0/2/0)

26. The equation of a circle with its centre at the origin and radius 1 is
$x^2 + y^2 = 1$

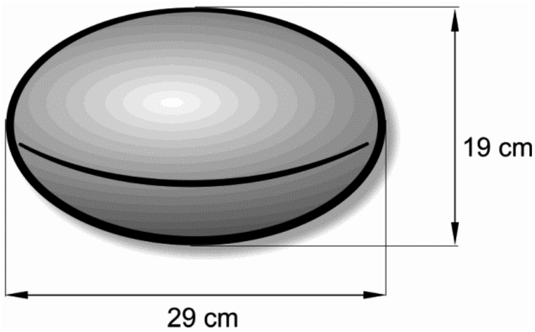
Similarly, the equation of an ellipse with its centre at the origin and that intersects the axis at $(\pm a, 0)$ and $(0, \pm b)$ is

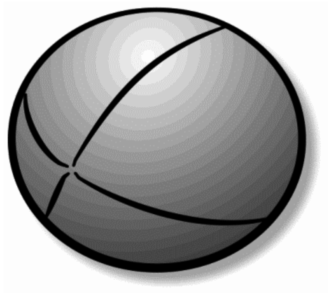
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$



When this type of ellipse rotates around the *x*-axis one gets an ellipsoid. The ball used in rugby has the shape of an ellipsoid.

One type of ball approved for rugby games has the measures given in the figure below.



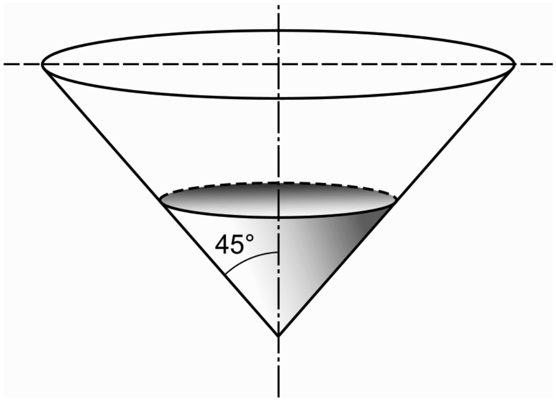


Calculate the volume of this ball. (0/0/3)

27. Lasse and Marcus are going to solve the following problem:

A container has the shape of a cone as can be seen from the figure. The container is initially empty. Water is added at a velocity of $(25+0.2t)$ litres/min, where t is the time in minutes from the start of the filling.

At what speed does the water level of the container raise when it is 7.0 dm?



Lasse first calculates that it takes 13.6 minutes for the water level to reach 7.0 dm.

Marcus will use Lasse's results to solve the rest of the problem. Marcus starts by denoting the water level with h and determines the volume of the container expressed in h. He then calculates the requested velocity.

a) Start with Lasse's results and perform Marcus's part of the solution. (0/1/1)

b) Show that Lasse's calculations are correct, that is, that the water level is 7.0 dm after 13.6 minutes. (0/1/2)